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Introduction

This Teacher’s Guide will assist you in interpreting and presenting the activities in the accompanying Student’s Book. It begins with an introduction to the Revised Junior Secondary School syllabi.

Introduction to the Revised Junior Secondary School syllabi

• The Revised Junior Secondary programme has not been drastically changed.
• The revised syllabi aim to convey the ideals reflected in the Revised National Policy on Education as well as Vision 2016.
• They are still based on the ten-year basic education philosophy and equip students with knowledge and skills that are relevant in today’s world.
• Students are encouraged to excel within their own capabilities. Teachers are encouraged to use participatory teaching and diverse learning approaches as individual talents, needs and learning styles are recognised.
• This inclusive approach urges teachers to accommodate all children regardless of their physical, intellectual, social, emotional, linguistic or other conditions.
• The syllabi also focus on the inculcation of attitudes and values that need to be nurtured.
• Emerging issues help students to understand and cope with the challenges and developments happening around them. These emerging issues, such as environmental education, HIV/AIDS education, gender equality, and the world of work, are infused into the syllabi.
• Assessment is a crucial part of the teaching and learning process, and should also consider those students with special needs.

Features of the Student’s Book

• The Student’s Book is divided into chapters. The first page of each chapter lists the objectives that will be covered in the chapter.
• At the start of the chapter, you will also find a list of key skills that students will acquire while engaging with the activities.
• Activities are used to explore, experiment, research and discover, while exercises reinforce the work the students have learnt.
• We have provided additional activities for students of mixed abilities. There are additional activities for students who need additional stimulation. They are called extension activities. There are also activities for students who need extra time to reflect on concepts, and practise what they have learnt. They are called support activities.
• Emerging issues boxes link current social and economic issues to the environment of students and make the learning more practical.
• There are also case studies, investigations and homework exercises to reinforce learning.
• Summaries help the students reflect on the work they have learnt.
- Revision exercises test their understanding of the work covered.
- Assessments provide exam-style questions for more practice.

**Features of the Teacher’s Guide**

The Teacher’s Guide is designed to be easy to use. It is divided into chapters (as per the Student’s Book) and contains information for each chapter. The Teacher’s Guide has the following features:

- At the start of each chapter, all the General and Specific Objectives to be covered in that chapter are listed.
- All the answers to the questions in the activities, exercises, revision and assessment sections in the Student’s Book appear in these sections.

**The importance of revision**

At the start of each chapter, spend time revising what students already know about the topic. The introductory activities in the Student’s Book will help you to test prior knowledge, and create an interest in the work to come. Before beginning any lesson, spend a few minutes revising the previous lesson. Use the revision sections at the end of each chapter, or part thereof, to give students opportunities to revise their work. This will allow you to gauge what gaps there are in their knowledge, and where additional help is needed.

**The importance of assessment**

Assessment measures whether students have achieved the identified objectives of a teaching and learning process. Assessment allows teachers to monitor progress, provide feedback and guidance, and diagnose barriers to learning. Simple observation in the classroom, peer-and self-evaluation, testing and project work all play a role in assessment. The key principle regarding assessment is that of continuous assessment. Teachers and students should reflect on the learning processes at appropriate times, and assess their own strengths and weaknesses. These comments should be recorded.

At various points in each term you will do formative assessment of relevant objectives, and give the students feedback to help them improve their performance. Towards the end of each term you will need to conduct a summative assessment. This will provide an overall picture of each student’s progress at a given time. Here is a summary of important tips for teachers:

1. Build on what your students already know.
2. Use materials that are meaningful, clear and interesting.
3. Allow students to master simple concepts first, and then apply these concepts to more complex ones.
4. Accurately assess your students so that you can plan better.
5. Give your students positive feedback so that they are motivated.
Syllabus: General objective

This chapter deals with numbers and number concepts. Students must have a clear understanding of all the different types of numbers.

Specific objectives

- Classify numbers into the categories in which they belong.
- Use a calculator to deal with numbers.

Students have to develop the skills to do the four basic mathematical operations – multiply, divide, add and subtract – with very large and very small numbers in scientific notation and normal notation on their calculators. Students should know how to convert between the different modes and be able to interpret numbers in scientific notation and normal notation.

Help students develop a ‘feel’ for significant numbers. For example, the budget of Botswana is measured in billions of pula. This means that amounts of P500 or P1 000 do not have any significance when compared to such a large figure. However, values of P1.00 and P0.50 are significant when referring to, for example, the price of petrol.

Answers

**Activity 1.1** *(Student’s Book page 4)*

1. a) \(-4 \in \mathbb{Z}, \mathbb{Q}, \mathbb{R}\)
   b) \(\frac{1}{3} \in \mathbb{Q}, \mathbb{R}\)
   c) \(-2 \in \mathbb{Z}, \mathbb{Q}, \mathbb{R}\)

2. a) True
   b) False
   c) True

Note: \(\pi\) is a transcendental number, because it does not originate from an integer value. The value of \(\sqrt{13}\) is irrational because it is non-recurring and 13 is an integer. For the purpose of this book, students should regard \(\pi\) as irrational.
Extension Activity 1.1

1. True
2. False
3. True

Support Activity 1.1

1. a) 0.2
   b) 0.428571428...
   c) 2.666666666...
   d) 0.625
   e) 0.818181818...
2. a) 4
   b) 3.605551275...
   c) 3.141592654...
3. a) $\frac{1}{5}$, $\frac{5}{8}$ and $\sqrt{16}$
   b) $\frac{3}{7}$, $\frac{22}{3}$ and $\frac{9}{11}$
   c) $\sqrt{13}$ and $\pi$

Activity 1.2

1. a) 0.2
   b) 0.428571428...
   c) 2.666666666...
   d) 0.625
   e) 0.818181818...
   f) 0.111111111...
   g) 0.8
   h) 0.87
   i) 1.058823529...
   j) 0.944444444...
2. a) 4
   b) 3.605551275...
   c) 3.141592654...
   d) 1.570796327...
   e) 0.666666666...
   f) 2.5
3. a) $\frac{1}{5}$, $\frac{5}{8}$, $\frac{12}{15}$, $\frac{522}{600}$, $\sqrt{16}$, $\sqrt{\frac{4}{9}}$ and $\frac{55}{22}$
   b) $\frac{3}{7}$, $\frac{22}{3}$, $\frac{9}{11}$, $\frac{1}{9}$, $\frac{18}{17}$, $\frac{17}{18}$ and $\sqrt{\frac{4}{9}}$
   c) $\sqrt{13}$, $\pi$ and $\frac{\pi}{2}$
Support Activity 1.2  

(Student’s Book page 7)

1. \( \frac{1}{5} \in \mathbb{Q} \)
   \( \frac{3}{7} \in \mathbb{Q} \)
   \( \frac{2}{3} \in \mathbb{Q} \)
   \( \frac{5}{8} \in \mathbb{Q} \)
   \( \frac{9}{11} \in \mathbb{Q} \)
   \( \frac{1}{9} \in \mathbb{Q} \)
   \( \frac{12}{15} \in \mathbb{Q} \)
   \( \frac{522}{600} \in \mathbb{Q} \)
   \( \frac{18}{17} \in \mathbb{Q} \)
   \( \frac{17}{18} \in \mathbb{Q} \)
   \( \sqrt{16} \in \mathbb{Q} \)
   \( \sqrt{13} \in \mathbb{Q}' \)
   \( \pi \in \mathbb{Q}' \)
   \( \frac{\pi}{2} \in \mathbb{Q}' \)
   \( \sqrt{\frac{4}{9}} \in \mathbb{Q} \)
   \( \frac{55}{22} \in \mathbb{Q} \)

2. 

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<tr>
<th></th>
<th>R</th>
<th>Q</th>
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Activity 1.3

1. See Support Activity 1.2 above.

2. |   | R | Q | Q' | N | N₀ | Z | Undefined |
   |---|---|----|---|---|---|---|-----------|
   a) 5 | ✓ | ✓ | x | ✓ | ✓ | ✓ | x         |
   b) √5 | ✓ | x | ✓ | x | x | x | x         |
   c) 2/0 | x | x | x | x | x | x | ✓         |
   d) −4 | ✓ | ✓ | x | x | x | ✓ | x         |
   e) 1.123 | ✓ | ✓ | x | x | x | x | x         |
   f) π² | ✓ | ✓ | x | x | x | x | x         |
   g) 7/9 | ✓ | ✓ | x | x | x | x | x         |
   h) √9 | ✓ | ✓ | x | ✓ | ✓ | ✓ | x         |
   i) √−27 | ✓ | ✓ | x | x | x | ✓ | x         |
   j) (6−6)/6 | ✓ | ✓ | x | x | x | ✓ | x         |
   k) 4.5 | ✓ | ✓ | x | x | x | x | x         |
   l) −0.00098 | ✓ | ✓ | x | x | x | x | x         |
   m) 0.012301230123 | ✓ | ✓ | x | x | x | x | x         |
   n) 3.1415926535… | ✓ | x | ✓ | x | x | x | x         |

3. No. 22/7 is only an approximation of the value of π. π is irrational, so we cannot express it as a fraction in the form Z/Z.

Exercise 1.1

1. a) 1 1/9
   b) 4/9
   c) 3 287/999
   d) 5/9
   e) 100 10/99
   f) 100 101/999
   g) 100 1 001/999
   h) 4 7 331/9 999

2. Write the fraction without the decimal point: 1 239. Subtract 1 from this number to get 1 238. Divide 1 238 by 999 to get the original decimal fraction of 1.239.
Chapter 1: Numbers

Homework Exercise  
(Student’s Book page 10)

2. Rational. The \( \pi^2 \) above and below the fraction line cancel each other out, which leaves a fraction of \( \frac{3}{7} \), which is rational because it takes the form \( \frac{Z}{Z} \).

Activity 1.4  
(Student’s Book page 11)

Extension Activity 1.2  
(Student’s Book page 12)

1. Associative Law
2. \( e = 1 \)
3. Yes

Exercise 1.2  
(Student’s Book pages 14)

1. 18
2. \(-6\)
3. 11
4. 10.25

Homework Exercise  
(Student’s Book page 14)

1. \( 1234 \times 8 + 4 = 9876; 12345 \times 8 + 5 = 98765; \ldots \)
2. \( 9876 \times 9 + 4 = 88888; 98765 \times 9 + 3 = 888888; \ldots \)
3. \( 1111 \times 1111 = 1234321; 11111 \times 11111 = 123454321; \ldots \)

Homework Exercise  
(Student’s Book page 16)

1. a) Exact area = 4.41 m\(^2\); approximate area = 4 m\(^2\); percentage error \( \approx 9.3\% \)
   b) Exact area = 91 264.41 m\(^2\); approximate area = 91 204 m\(^2\); percentage error \( \approx 6.6\% \)
   c) The larger the original value, the smaller the percentage error.
Exercise 1.3

(Student’s Book page 16)

1. a) 4.97
   b) 0.24
   c) −1.11
   d) 1.01
   e) 10.25

Extension Activity 1.3

(Student’s Book page 17)

0.557

Revision Exercise

(Student’s Book page 17)

1. a) Irrational
   b) Rational
   c) Rational
   d) Rational
   e) Rational
   f) Irrational

2. a) \(0.9 \overline{7} = \frac{8.8}{9} = \frac{88}{90} = \frac{44}{45}\)
   b) \(0.12 \overline{7} = 0.127127127\ldots = \frac{127}{999}\)
   c) \(5.\overline{3} = 5.353535\ldots = \frac{530}{99}\)
   d) \(0.00\overline{1} = 0.00111\ldots = \frac{111}{99900}\)

3. a) \(0.\overline{8}\)
   b) \(1.\overline{3}\)
   c) \(5.\overline{7}\)
   d) \(0.\overline{0}\)

4. a) True
   b) False (Fractions – common and decimal – and irrational numbers are also real numbers.)
   c) True
   d) False (\(\sqrt{16} = 4 \in \mathbb{Q}\))
   e) False (\(\pi\) is an irrational and real number, but not a natural number.)
   f) False (\(\frac{0}{0}\) is undefined.)
   g) True
   h) False (\(7 < \sqrt{50} < 8\))
   i) False (A prime number has two different factors, namely, 1 and itself.)
   j) False (All mixed fractions are rational.)

5. a) \(\approx 1.806\) or \(\approx 1.782\)
   b) \(\approx 0.978\)
   c) \(\approx 3.785\)
Chapter 2: Patterns and sequences

Syllabus: General objective

This chapter deals with number patterns and sequences. Students must be able to identify number patterns, and make connections between patterns in nature and arithmetic sequences.

Specific objectives

- Describe in words the pattern of a linear sequence.
- Express the pattern of a linear sequence in algebraic form.
- Generate a sequence of numbers.
- Use the patterns of sequences to represent given situations and use them to solve real-life problems.

Teach students the skill of looking for number patterns or patterns that are mathematically significant. Remind students to be aware of the world around them and that mathematical patterns appear frequently.

Answers

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<th>Activity 2.1</th>
<th>(Student’s Book page 21)</th>
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<tr>
<td>1. ( T_5 = 11 ) Sequence starts with 3. Add 2 to the previous term.</td>
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<tr>
<td>2. ( T_5 = 8 ) Sequence starts with (-4). Add 3 to the previous term.</td>
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<tr>
<td>3. ( T_5 = -9 ) Sequence starts with 7. Subtract 4 from the previous term.</td>
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<td>4. ( T_5 = 25 ) Sequence starts with 1. Square the number of the term.</td>
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<tr>
<td>5. ( T_5 = -25 ) Sequence starts with (-5). Subtract 5 from the previous term.</td>
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<tr>
<td>6. ( T_5 = 3 ) 125 Sequence starts with 5. Multiply the previous term by 5.</td>
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<tr>
<td>7. ( T_5 = -32 ) Sequence starts with (-2). Multiply the previous term by (-2).</td>
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<tr>
<td>8. ( T_5 = -75 ) Sequence starts with (-3). Multiply (-3) by the square of the number of the term.</td>
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Activity 2.2  (Student's Book page 21)

Patterns 1 (+2), 2 (+3), 3 (−4) and 5 (−5) are linear.

Exercise 2.1  (Student's Book page 22)

1. Linear. Add −0.3 to the previous term.
2. Not linear.
4. Linear. Add 2 to the previous term.
5. Not linear.
6. Linear. Add −3.5 to the previous term or subtract 3.5.

Activity 2.3  (Student's Book page 23)

1. \[ T_n = -3 + (n - 1) \times 3 = -3 + 3(n - 1) = -3 + 3n - 3 = 3n - 6; \]
   \[ T_{10} = 3(10) - 6 = 30 - 6 = 24 \]
2. \[ T_n = n - 1 \frac{1}{2}; \]
   \[ T_{10} = 8 \frac{1}{2} \]
3. \[ T_n = 0.3n - 1; \]
   \[ T_{10} = 2 \]
4. \[ T_n = 6 - 2n; \]
   \[ T_{10} = -14 \]

Exercise 2.2  (Student's Book page 23)

1. Sequence starts with −14. Add 14 to the previous term; \[ T_n = 14n - 28; \]
   \[ T_8 = 84 \]
2. Sequence starts with 7. Add 2 to the previous term; \[ T_n = 2n + 5; \]
   \[ T_8 = 21 \]
3. Sequence starts with 8. Subtract 3 from the previous term; \[ T_n = 1 - 3n; \]
   \[ T_8 = -13 \]
4. Sequence starts with −4. Subtract 5 from the previous term; \[ T_n = 1 - 5n; \]
   \[ T_8 = -39 \]
5. Sequence starts with \( \frac{1}{3} \). Add \( \frac{1}{3} \) to the previous term; \[ T_n = \frac{1}{3}n; \]
   \[ T_8 = \frac{22}{3} \]
6. Sequence starts with \( \frac{1}{4} \). Subtract \( \frac{1}{2} \) from the previous term; \[ T_n = \frac{13}{4} - \frac{1}{2}n; \]
   \[ T_8 = -\frac{21}{4} \]

Support Activity 2.1  (Student's Book page 24)

1. \( T_{20} = 40; \)
   \( T_{50} = 100 \)
2. 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; ...
3. \( T_{50} = 160 \)

Activity 2.4  (Student's Book page 25)

1. a) 2; 5; 10; 17  b) 3; 7; 11; 15  c) 33; 34; 35; 36
2. a) 401  b) 79  c) 52
3. b) and c)
Exercise 2.3  (Student's Book page 25)

1. $-3; -1; 1; T_{10} = 15$ (linear)
2. $1; -3; -7; T_{10} = -35$ (linear)
3. $12; 13; 14; T_{10} = 21$ (linear)
4. $-7\frac{1}{2}; -8; -8\frac{1}{2}; T_{10} = -12$ (linear)
5. $4; 6.5; 9; T_{10} = 26.5$ (linear)
6. $0; 6; 16; T_{10} = 198$

Exercise 2.4  (Student's Book page 27)

1. a) $T_n = 13 + 5n$
   b) $T_{40} = 213$
2. a) $T_{30} = 100$
   b) $T_{50} = 180$, so there are 50 terms ($n = 50$).

Extension Activity 2.1  (Student’s Book page 27)

1. $T_n = n^2 \times (-1)^n$
2. $T_n = \frac{n}{(n + 1)}$
3. $T_n = (n^2)(3^n)$
4. $T_n = 4n - 7$
5. $T_n = \frac{(2n - 1)}{(3n - 2)}$

Exercise 2.5  (Student’s Book page 28)

1. a) $d = 1\frac{1}{2}$
   b) $T_n = 1\frac{1}{2}n + 3\frac{1}{2}$
   c) $T_{100} = 153\frac{1}{2}$
   d) $n = 150$
2. $3; 7; 11$
3. a) 96 is not a term in this sequence.
   b) $T_n = 6n + 10$
      $200 = 6n + 10$
      $\therefore n = 31.6$
      $\therefore$ 200 is not a term in this sequence because $n \in \mathbb{N}$.
4. a) $T_{41} = 18.7$
   b) $T_4 = 0.2$ $\therefore T_4$ is closest to zero.
   c) $T_{77} = 36.7$
   d) 23 terms are less than 10.
Extension Activity 2.2  
(Student’s Book page 28)

1. The first difference for this sequence is 2; 1; 0; −1; …. Therefore, the constant second difference is −1; −1; −1; −1; …. because the pattern in the first difference is to subtract 1 from the previous term.

2. \( T_1 = a(1)^2 + b(1) = 3 \)
   \( T_2 = 4a + 2b = 5 \)
Solve simultaneously:
   \( a = -\frac{1}{2}; \ b = 3.5 \)
∴ \( T_n = -\frac{1}{2}n^2 + 3.5n \)

3. \( T_6 = 3; \ T_7 = 0; \ T_{10} = -15 \)

4. No, the general term shows us that this is a quadratic sequence.

Exercise 2.6  
(Student’s Book page 30)

1. a) 17 logs
   b) 10 rows

2. a) 15 stones
   b) 9 rows

3. \( T_n = 25n + 25 \)
   \( \therefore 350 = 25n + 25 \)
   \( \therefore n = 13 \)
∴ They will swim 350 m in the 13th session.

4. a) P2 200
   b) Year 7

5. a) \( A = 660n + 6000 \)
   b) \( T_2 = P8 \ 640; \ T_5 = P9 \ 300; \ T_6 = P9 \ 960; \ T_7 = P10 \ 620; \ T_8 = P11 \ 280 \)
   c) \( T_{20} = P19 \ 200 \)
   d) 35 years

Investigation  
(Student’s Book page 32)

1. \( T_1 = 1^2 - 1 + 11 = 11 \) \( T_4 = 23 \)
   \( T_2 = 2^2 - 2 + 11 = 13 \) \( T_5 = 31 \)
   \( T_3 = 3^2 - 3 + 11 = 17 \)
Yes, the conjecture seems to be correct, because these five numbers are all prime numbers.

2. The first five terms are prime numbers, but not all the prime numbers between 11 and 31 are generated: 19 and 29 are missing.

3. The conjecture does not hold water for all values of \( n \) because \( n = 11 \) generates a perfect square. Using a spreadsheet we find that, among the first 40 terms, the following terms are not prime: \( T_{11}; \ T_{12}; \ T_{15}; \ T_{23}; \ T_{28}; \ T_{33}; \ T_{34}; \ T_{36}; \ldots \).
Extension Activity 2.3  
(Student’s Book page 34)

1. 13; 6; −1; −8; ...
2. c)

Revision Exercise  
(Student’s Book page 35)

1. a) No, because substituting the value of \( n \) into the rule does not give the value of the number at that position in the sequence.
   b) \( T_n = 3n + 3 \)
   c) \( T_{100} = 303 \)
2. a) 2; 1; 0; −1; −2
   b) 7; 9; 11; 13; 15
   c) −2; 3; 8; 13; 18
   d) \( 1\frac{1}{4}; 1\frac{3}{4}; 2\frac{1}{4}; 2\frac{3}{4}; 3\frac{1}{4} \)
   e) 14; 33; 52; 71; 90
   f) −8; −11; −14; −17; −20
3. a) 13; 15; 17; \( T_n = 2n + 3 \); \( T_{50} = 103 \)
   b) −11; −15; −19; −23; −27; \( T_n = 9 - 4n \); \( T_{50} = -191 \)
   c) 21; 25; 29; 33; 37; \( T_n = 4n + 1 \); \( T_{50} = 201 \)
   d) 0.3; 0; −0.3; −0.6; −0.9; \( T_n = 1.8 - 0.3n \); \( T_{50} = -13.2 \)
   e) 275; 350; 425; 500; 575; \( T_n = 75n - 25 \); \( T_{50} = 3725 \)
   f) \( -1\frac{3}{4}; -3\frac{1}{4}; -1\frac{1}{4}; 1\frac{1}{4}; T_n = 0.5n - 3\frac{1}{4}; T_{50} = 21\frac{3}{4} \)
4. a) \( T_n = 5n + 1 \)
   b) \( T_{200} = 1001 \)
   c) 120 hexagons
5. a) +4
   b) 4; 8; 12; ...
   c) There is a difference of 4 between terms in the same position.
   d) \( T_n = 4n + 3 \)
6. a) P7 000; P6 550; P6 100; ...
   b) P5 200
   c) 14 years
7. a) 8 000; 7 966; 7 932; 7 898; 7 864
   b) 119 days
Syllabus: General objective

This chapter deals with indices. Students must acquire knowledge on indices and their role in real-life problems.

Specific objectives

- Interpret integral and positive fractional indices.
- Simplify expressions that contain indices.
- Solve problems that involve integral and positive fractional indices.
- Order numbers – in ascending order and descending order – expressed in standard form.
- Solve problems that involve numbers expressed in standard form.

Teach students how to write and work with powers of numbers. Ensure that they can use exponential notation to write very large or very small numbers in a more convenient way.

Answers

**Activity 3.1** *(Student’s Book page 39)*

1. $x^4$
2. $d^{10}$
3. $x^3y^7$
4. $x^9y^{21}$
5. $8a^9$
6. $81x^{12}$
7. $-27x^{12}$
8. $8a^9b^{15}c^{12}$
9. $\frac{x}{4}$
10. $-24t^{13}$
11. $a^4$
12. $\frac{b^c}{a^2}$
Homework Exercise  
(Student’s Book page 40)
Yes. \((2^2)^3 = 64\), but \(2^{2^3} = 2^8 = 256\). The correct answer is 64.

Support Activity 3.1  
(Student’s Book page 41)
1. 3  
2. 10  
3. 8  
4. 5

Activity 3.2  
(Student’s Book page 41)
1. a) 3  
b) 10  
c) \(\frac{4}{9}\)  
d) 2  
e) 10  
f) 4  
g) 1000  
h) 8  
i) 5
2. a) Undefined  
b) Defined: \(\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2\)  
c) Defined: \(\sqrt[3]{-16} \approx -2.52\)
3. a) False: \(\sqrt{9} + 16 = \sqrt{25} = 5 \neq 3 + 4\); square and add first, then find the square root.  
b) True: \(\sqrt{9} + 16 = \sqrt{144} = 12 = 3 \times 4\)

Exercise 3.1  
(Student’s Book page 42)
1. a) \(\frac{x^{\frac{1}{2}}}{5}\)  
b) \(a^{\frac{3}{3}}\)  
c) \(6x^4\)  
d) \(2a^3b^2\)  
e) \(3^{9x} \text{ or } 27^{3x}\)  
f) Stays unchanged.
2. a) \(\sqrt[3]{a^2}\)  
b) \(3\sqrt[3]{x^3}\)  
c) \(4x^2\)  
d) \(a^2\)  
e) \(3\sqrt[3]{x^2}\)  
f) 8

Exercise 3.2  
(Student’s Book page 43)
1. a) 27  
b) 32  
c) \(32a^{10}\)  
d) \(9a^2\)  
2. a) \(\sqrt[3]{4}; \sqrt[4]{3}\)  
b) \(\sqrt[3]{7}; \sqrt[4]{12}; \sqrt[3]{3}\)
Activity 3.3

1. The size of the flea population doubles every five days. Therefore, the exponent should be expressed as a multiple of 5.
2. 240 fleas
3. 20 days

Activity 3.4

1. Growth = P10 000 + P10 000 × $\frac{14.5}{100}$
   = P10 000(1 + 0.145)
   = P10 000 × 1.145
   ∴ Penny’s money is multiplied by 1.145 each year.
2. P10 000
3. a) $A = P10 000 \times 1.145^2$
   = P13 110.25
b) $A = P10 000 = 1.145^5$
   = P19 680.11
c) $A = P10 000 = 1.145^{10}$
   = P38 730.66
4. No, the investment’s value after 5 years is P19 680.11. Therefore, the investment will double in the 6th year.
5. $P10 000 \times 1.145^n = P338 000$
   ∴ $1.145^n = 33.8$
   ∴ $n \approx 26$ years

Exercise 3.3

1. a) The number of bacteria in the culture doubles every minute.
   b) 180 bacteria
   c) 
<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 180 \times 2^t$</td>
<td>180</td>
<td>360</td>
<td>720</td>
<td>1 440</td>
<td>2 880</td>
<td>5 760</td>
</tr>
</tbody>
</table>
   d) $180 \times 2^t > 40 000$
   ∴ $2^t > 222.22$
   ∴ $t > 7.8$
   ∴ $t \approx 8$ min
2. a) $100% – 5.5% = 94.5\%$, which represents the volume of air that remains after each day.
   b) $V_{air} = 4 200 \times \left(\frac{94.5}{100}\right)^t$
   = $4 200 \times 0.945^t$, where $t$ is the number of days
   c) 2 671.18 cm³
d) \[4200 \times 0.945^t < 1000\]
\[\therefore t > 25.4\]
\[\therefore t \approx 26\text{ days}\]

**Note:** Students should use the trial-and-error method. The inequality sign changes around because the bases on either side of the equation are fractions. The following example may help students to understand it better:

If \[\left(\frac{1}{2}\right)^t < \left(\frac{1}{4}\right)^t\], then \(\left(\frac{1}{2}\right)^t < \left(\frac{1}{2}\right)^2\). This is possible only for \(t > 2\).

3. a) \[P = 2000000 \times (1 + \frac{1.9}{100})^t\]
   \[= 2 \times 10^6 \times 1.019^t\]
   b) \[P = 2 \times 10^6 \times 1.019^t\]
   \[\approx 2197358\text{ people}\]
   c) \[1.019^t = 1.1\]
   \[\therefore t \approx 5.1\text{ years}\]

**Project**

*(Student’s Book page 45)*

1. \[\frac{4}{2500} \times \frac{100}{1} = 0.16\%\]
   On the first day, this doesn’t seem much, but he should be concerned because the plant grows exponentially (the next day it covers double the surface area than the previous day).

2. It is exponential, because the growth doubles every day from the day before, not from the first day. \(A = 4 \times 2^{-t}\), where \(t\) is the number of days starting from 1 April. Alternatively,
   \(A = 2 \times 2^t = 2^{t-1}\), where \(t\) is the number of days starting from 1 April.

3. 

4. \(t \approx 9.3\) days (determined from the graph), so on 9 April it will cover half.

5. \(t \approx 10.3\) days (determined from the graph), so on 10 April it will cover the dam completely.

6. Kill all the plants before 3 April, when it will still be manageable (16 m²).
Support Activity 3.2  
(Student’s Book page 46)

1. a) $7.89 \times 10^5$
   b) $2.0785 \times 10^6$
   c) $2.84 \times 10^{-7}$
   d) $5 \times 10^{-7}$
2. a) 23 000 000
   b) 0.000752

Exercise 3.4  
(Student’s Book page 47)

1. a) $4.56 \times 10^5$
   b) $8.09 \times 10^6$
   c) $9.89 \times 10^{-7}$
   d) $1 \times 10^{-7}$
   e) $4.50 \times 10^{-5}$
   f) $1.23 \times 10^8$
   g) $4.35 \times 10^{-1}$
   h) $2 \times 10^9$
2. a) 87 000 000
   b) 23 400
   c) 0.0000000275
   d) 0.0004

Activity 3.5  
(Student’s Book page 47)

1. $7.5 \times 10^3 > 1.22 \times 10^3 > 7.05 \times 10^2 > 3.45 \times 10^{-2} > 4.53 \times 10^{-3}$
2. $9.34 \times 10^{-3} < 8.76 \times 10^{-2} < 9.8101 \times 10^1 < 2.0237 \times 10^2 < 2.35098 \times 10^2 < 3.4 \times 10^6$

Exercise 3.5  
(Student’s Book page 48)

1. a) $7.5 \times 10^8$
   b) $2.6 \times 10^3$
   c) $2.1 \times 10^{-5}$
   d) $3.4 \times 10^{11}$
2. a) $2.077 \times 10^9$
   b) $1.152 \times 10^{-15}$
   c) $7.651 \times 10^8$
   d) $2.736 \times 10^{-2}$

Activity 3.6  
(Student’s Book page 48)

1. 319 450 or $3.1945 \times 10^5$ people
2. 122 988 or $1.22988 \times 10^5$ people
3. Gaborone: \( P = 1.86 \times 10^5 \times (1 + \frac{0.85}{100})^9 \)
   \[ = 2.007 \times 10^6 \text{ people} \]

   Mahalapye: \( P = 1.0445 \times 10^5 \times (1 + \frac{0.85}{100})^9 \)
   \[ = 1.127 \times 10^5 \text{ people} \]

   Lobatse: \( P = 2.9 \times 10^4 \times (1 + \frac{0.85}{100})^9 \)
   \[ = 3.130 \times 10^4 \text{ people} \]

### Exercise 3.6

(\text{Student’s Book page 49})

1. \( 2 \times 10^6 - 1.785 \times 10^6 \)
   \[ = 2.15 \times 10^6 \text{ people} \]
   \[ = 215 \text{,}000 \text{ people} \]
2. \( P = 1.785 \times 10^6 \times (1 + \frac{0.85}{100})^9 \)
   \[ = 1.926 \times 10^6 \text{ people} \]
   This is not accurate, because the answer should be \( 2 \times 10^6 \). The growth rate was actually closer to 1.27%. Find this value by the following calculation:
   \[ 2 = 1.785(1 + i)^9 \]
   \[ i = \sqrt[9]{\frac{2}{1.785}} - 1 \]
   \[ \approx 0.0127 \]
   \[ \therefore r \approx 1.27\% \]
3. Number of births = \( \frac{24.8}{1000} \times (2 \times 10^6) \)
   \[ = 49 \text{,}600 \text{ births} \]

   Number of deaths = \( \frac{13.9}{1000} \times (2 \times 10^6) \)
   \[ = 27 \text{,}800 \text{ deaths} \]
4. \( \text{CO}_2 \text{ emissions} = (2 \times 10^6) \times 2.4 \)
   \[ = 4.8 \times 10^6 \text{ units of } \text{CO}_2 \]
5. Required total calories = \( (2 \times 10^6) \times (2.16 \times 10^3) \)
   \[ = 4.32 \times 10^9 \text{ calories} \]
6. Number of people who do not have water = \( 0.3 \times (2 \times 10^6) \)
   \[ = 6 \times 10^5 \text{ or } 600 \text{,}000 \text{ people} \]

### Extension Activity 3.1

(\text{Student’s Book page 50})

1. a) Population density = \( \frac{(2 \times 10^6)}{(5.696 \times 10^5)} \)
   \[ = 3.5 \text{ people/km}^2 \]

   b) Arable land = \( (5.696 \times 10^5) \times 0.02 \)
   \[ = 1.1392 \times 10^4 \text{ km}^2 \]
   \[ = 11392 \text{ km}^2 \]
Grassland = \((5.696 \times 10^5) \times 0.58\)
\[= 3.30368 \times 10^5 \text{ km}^2\]
\[= 330\ 368 \text{ km}^2\]
Wooded area = \((5.696 \times 10^5) \times 0.19\)
\[= 1.08224 \times 10^6 \text{ km}^2\]
\[= 108\ 224 \text{ km}^2\]
Arid land and desert = \((5.696 \times 10^5) \times 0.15\)
\[= 8.444 \times 10^4 \text{ km}^2\]
\[= 85\ 440 \text{ km}^2\]
c) Total degraded area = \((2.85 \times 10^4 \text{ km}^2) + (3.516 \times 10^4 \text{ km}^2) + (3.71 \times 10^4 \text{ km}^2)\)
\[= 1.0076 \times 10^5 \text{ km}^2\]
\[= 100\ 760 \text{ km}^2\]

Percentage degraded area = \(\frac{1.0076 \times 10^5 \text{ km}^2}{(5.696 \times 10^5)} \times \frac{100}{1}\)
\[= 0.177 \times 100\]
\[= 17.7\% \text{ degraded area}\]

2. a) Difference = \((127 \times 10^4 \text{ km}) - (6.79 \times 10^3 \text{ km})\)
\[= 1.26 \times 10^6 \text{ km}\]
b) \((127 \times 10^4 \text{ km}) + (6.79 \times 10^3 \text{ km}) = 1.28 \times 10^6 \text{ km}\)
c) Magnitude difference = \(\frac{127 \times 10^4 \text{ km}}{(6.79 \times 10^3 \text{ km})}\)
\[\approx 187 \text{ times}\]

**Revision Exercise**

(Student’s Book page 51)

1. a) \(-2xy^8\)  b) \(9x^5\)

c) \(x^9\)  d) \(8\)

e) \(8a^6\)  f) \(x^6\)

g) \(1\ 024\)  h) \(32\)

i) \(16.5\)  j) \(100x^6y^2\)

k) \(15.625\)  l) \(\frac{36x}{1x^{-2}}\)

m) \(2\)  n) \(\frac{x^5}{y^3}\)

o) \(x^\frac{1}{y^\frac{3}{5}}\)
2. a) 3 600  
b) 9 815 000  
c) 0.004276  
d) 0.0000678  
3. a) $6 \times 10^4$  
b) $1.76 \times 10^{-5}$  
c) $5.79 \times 10^3$  
d) $7.2 \times 10^{-5}$  
4. a) $6.66 \times 10^8$  
b) 440.8  
c) 15.76  
d) 1 174.24  
5. a) 60 s  
b) 3 600 s or $3.6 \times 10^3$ s  
c) 86 400 s or $8.64 \times 10^3$ s  
d) 604 800 s or $6.048 \times 10^5$ s  
e) 365-day year: 31 536 000 s or $3.1536 \times 10^7$ s  
Leap year: 31 622 400 s or $3.16224 \times 10^7$ s  
f) There are 3 or 4 leap years in any 15-year period.  
∴ Number of seconds in 15 years = $3(3.16224 \times 10^7$ s) + $12(3.1536 \times 10^7$ s)  
= $4.73992 \times 10^8$ s  
OR  
= $4(3.16224 \times 10^7$ s) + $11(3.1536 \times 10^7$ s)  
= $4.733856 \times 10^8$ sec  
6. a) i) $\left(3 \times 10^6\right) \times 60 \times 5$  
\[= 9 \times 10^{10} \text{ m}\]  
ii) $\left(3 \times 10^6\right) \times 60 \times 60 \times 24 \times 7 \times 8$  
\[= 1.45152 \times 10^{15} \text{ m}\]  
iii) $\left(3 \times 10^6\right) \times 60 \times 60 \times 24 \times 365.25$  
\[= 9.46728 \times 10^{15} \text{ m}\]  
b) Time = $\frac{\text{distance}}{\text{speed}}$  
\[= \frac{\left(1.5 \times 10^{11} \text{ m}\right)}{\left(3 \times 10^8 \text{ m.s}^{-1}\right)}\]  
\[= 500 \text{ s}\]  
\[= 8.33 \text{ min}\]  
\[= 8 \text{ min 20 s}\]  
7. a) 150 000 = the volume of water that is let out at any given time.  
\[
\frac{d}{10} = 100\% - 0.6\% = 99.4\%
\]  
\[
\text{the number of 10-km stretches, because water evaporates at a rate of 0.6\% per 10-km stretch.}
\]  
b) Yes, because the expression is raised to the power of $\frac{d}{10}$.  
c) 150 000 $\ell$  
d) $V = 150 000 \times 0.994^{15}$  
\[\approx 137 053 \ell\]  
e) You can either cover the channel or let water out during the night only.
Syllabus: General objective

This chapter deals with depreciation, appreciation and compound interest.

Specific objectives
- Solve problems that involve depreciation, appreciation and compound interest.
- Understand the contexts in which these concepts are applicable.
- Learn about inflation and the effect it has on the price of basic commodities.
- Differentiate between compound interest and simple interest.

Students must apply what they learnt about indices in Chapter 3. They should be able to use this knowledge to calculate compound interest, depreciation and appreciation. Demonstrate how effective and useful a calculator is through the calculations in this chapter.

Answers

Support Activity 4.1 (Student’s Book page 56)

1. a) $100 \times 0.08$ represents 8% interest. This 8% added to 100, gives the new value of P108.
   b) $1.08 = 1 + 0.08$, where 1 represents 1 whole (the principal amount of P100) and 0.08 represents the 8% interest. Multiplying the principal amount of P100 by 1.08 gives the new value of P108.
   Method b) is quicker.
2. a) 1.1
   b) 1.32
   c) 1.032

Exercise 4.1 (Student’s Book page 56)

1. $A = P500(1 + \frac{6.2}{100} \times 5)$
   = P655
   $\therefore$ Interest = P655 – P500
   = P155
2. \( P3 400 = P(1 + \frac{8}{100} \times 6) \)
   \[ \therefore P = P2 297.30 \]
3. \( P620 = P550(1 + i \times 2) \)
   \[ \therefore i \approx 0.0636 \quad \approx 6.36\% \]
4. \( P + P440 = P(1 + 0.078 \times 3.5) \)
   \[ 0.273P = 440 \]
   \[ P = P1 611.72 \]

**Activity 4.1**

(Student’s Book page 58)

1. Compound interest is an exponential function where simple interest is a linear or straight-line function.
2. \( A \) = the accumulated amount at the end of the investment period
   \( P \) = the principal amount initially invested
   \( i \) = the interest rate as a %
   \( n \) = the period, or number of accumulations
3. a) \( i \) changes to \( \frac{i}{2} \), so \( i = \frac{0.08}{2} \).
   \( n \) changes to \( n \times 2 \).
   b) \( i \) changes to \( \frac{i}{4} \), so \( i = \frac{0.08}{4} \).
   \( n \) changes to \( n \times 4 \).
   c) \( i \) changes to \( \frac{i}{12} \), so \( i = \frac{0.08}{12} \).
   \( n \) changes to \( n \times 12 \).
   d) \( i \) changes to \( \frac{i}{365} \), so \( i = \frac{0.08}{365} \).
   \( n \) changes to \( n \times 365 \).
4. The formula for 3d) gives the highest yield because the interest accumulates daily.

**Exercise 4.2**

(Student’s Book page 59)

1. a) Interest = \( P10 000 \times \frac{8}{100} \)
   \[ = P800 \]
   Total interest for 10 years = \( 10 \times P800 = P8 000 \)
   \[ \therefore A = P18 000 \]
   b) \( A = P10 000(1 + 0.08)^{10} \)
   \[ = P21 589.25 \]
   c) \( A = P10 000(1 + \frac{0.08}{12})^{10 \times 12} \)
   \[ = P22 196.40 \]
   d) \( A = P10 000(1 + \frac{0.08}{4})^{10 \times 4} \)
   \[ = P22 080.40 \]
2. \( P12 500 = P(1 + \frac{0.09}{12})^{5 \times 12} \)
   \[ \therefore P = P7 983 75 \]
3. Bank A: \[ A = 5000(1 + 0.1)^{10} \]
   \[= 12,968.71 \]
Bank B: \[ A = 5000(1 + \frac{0.0975}{12})^{120} \]
\[= 13,203.72 \]
Bank B is the better option because it yields P235.01 more than Bank A. However, point out to students that a difference of P235.01 over a 10-year period is not significant. Small differences in interest rates in this range only become significant when investing large sums of money.

4. a) \[ P_{220\,400} = P_{150\,000}(1 + i \times 5) \]
   \[\therefore i \approx 0.0939 \]
   \[\approx 9.39\% \]
   b) \[ P_{220\,400} = P_{150\,000}(1 + i)^5 \]
   \[\therefore i = 0.08 \]
   \[= 8\% \]
   c) \[ P_{220\,400} = P_{150\,000}(1 + \left(\frac{i}{12}\right)^{60}) \]
   \[\approx 0.0772 \]
   \[\approx 7.72\% \]

5. a) Option 1: P24 645.48
   Option 2: P24 812.29
   Option 3: P24 890.13
   b) Option 1 is the cheapest. Point out to students that a lower interest rate does not necessarily mean that it is cheaper, but that it also depends on the number of times the interest is compounded per annum.

Exercise 4.3  
(Student's Book page 62)

1. \[ P = \frac{P_{2\,355}}{1.12} \]
   \[= P2\,102.68 \]
   VAT = \[ P2\,102.68 \times \frac{12}{100} \]
   \[= P2\,52.32 \]

2. a) \[ A = P35\,240 \times 1.12 \]
   \[= P39\,468.80 \]
   b) \[ A = P39\,468.80 \times 1.3 \]
   \[= P51\,309.44 \]
   c) \[ A = P51\,309.44 \times (1 - 0.3) \]
   \[= P35\,916.61 \]
   d) Profit = \[ \frac{P35\,916.61}{1.12} - P35\,240 \]
   \[= -P3\,171.60 \]
   \[= P3\,171.60 \text{ loss} \]

Note: Point out to students that a mark-up of 30% followed by a discount by the same margin on the marked-up price results in a loss. This occurs because we first
multiply by 1.3 and then by 0.7, but 1.3 \times 0.7 \neq 1. Rather, 1.3 \times 0.7 = 0.91. This indicates a loss of 1 - 0.91 = 0.09, which is equivalent to 9\%.

e) \% loss = \frac{P3\ 171.60 \times 100}{P3\ 240} = 9\%

**Note:** This is the same result as that found in question d).

3. a) No, because a 20\% reduction would result in a price of:

   P120\ 000 \times 0.8 = P96\ 000.

   b) Purchase price = \frac{P100\ 000}{1.18} = P84\ 745.76

**Note:** It is essential that students understand the difference in calculation between finding the original price from a mark-up (Original price = \frac{marked-up price}{1 + \frac{percentage\ mark-up}{100}}) and reducing the price of an article by a certain percentage (reduced price = price \times (1 - \frac{percentage\ discount}{100})).

4. You can buy from either shop, because both values work out P1\ 008.

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**Exercise 4.4**  
*(Student’s Book page 65)*

1. a) \( A = P180\ 000(1 - 5 \times 0.13) \)
   \[ = P63\ 000 \]
   b) \( A = P180\ 000(1 - 0.13)^5 \)
   \[ = P89\ 715.77 \]

2. a) The computers depreciate at P10\ 000 per annum until their value is P0.
   b) \( \frac{P50\ 000}{P10\ 000} = 5 \text{ years} \)

**Note:** Point out that when an article depreciates according to the straight-line method, its value eventually becomes zero. In contrast, when an article depreciates according to the reducing-balance method, its value can never become zero. You can introduce the asymptotic effect here.

3. a) \( P800 = P(1 - 5 \times 0.15) \)
   \[ \therefore P = P3\ 200 \]
   b) \( P800 = P(1 - 0.15)^2 \)
   \[ \therefore P \approx P1\ 803 \]

4. a) \( P800 = P4\ 500(1 - 6 \times i) \)
   \[ \therefore i \approx 0.1962 \]
   \[ \approx 19.6\% \]
   b) \( P800 = P4\ 500(1 - i)^6 \)
   \[ \therefore i \approx 1 - \frac{P800}{P4\ 500} \]
   \[ \approx 25\% \]
5. \( P 8 \ 000 = P (1 - 0.15)^6 \)
   \[ \therefore P = P 21 \ 211.75 \]
6. \( A = P 40 \ 000 (0.88)^{10} \)
   \[ = P 11 \ 140.04 \]
7. Depreciation = \( \frac{P 270 \ 000}{9 \text{ years}} \)
   \[ = P 30 \ 000 \text{ per annum} \]
   Rate of depreciation = \[\frac{P 30 \ 000 \times \frac{100}{1}}{P 270 \ 000} \approx 11.1\%\]

**Extension Activity 4.1**

(Students’ Book page 65)

\( n = 12 \) years

**Exercise 4.5**

(Students’ Book page 66)

1. a) Selling price = \( P 850 \ 000 \times 1.16 \)
   \[ = P 986 \ 000 \]
   b) \( A = P 986 \ 000 (1 + 0.095)^5 \)
   \[ = P 1 \ 552 \ 199 \]
2. a) \( A = P 110 \ 000 (1 + 0.075)^{12} \)
   \[ \approx P 261 \ 996 \]
   b) \( A = P 20 \ 000 (1.06)^{12} \)
   \[ \approx P 40 \ 244 \]
   c) First year: P 40 244
      Second year: P 42 659
      Third year: P 45 218
      Total: P 128 121
      Phembi would have enough money.
3. **Note**: Appreciation on property is compounded annually.
   Property value = \( P 50 \ 000 (1.075)^{20} \)
   \[ \approx P 212 \ 393 \]
   Shares’ value = \( P 50 \ 000 (1 + 0.12 \times 20) \)
   \[ = P 170 \ 000 \]
   Precious should take the property option because the property investment at a lower interest rate is more profitable.
4. \( P 1 \ 000 \ 000 = P 222 \ 000 (1 + i)^{15} \)
   \[ \therefore i \approx 0.11259 \]
   \[ \approx 11.26\% \]
5. \( 2 = 1(1 + i)^8 \)
   \[ i \approx 0.0905 \]
   \[ \approx 9.05\% \]
6. \[ A = P1\, 500\, 000(1 + 0.089)^3 \]
   \[ = P1\, 937\, 202 \]

**Exercise 4.6**

(Student’s Book page 67)

1. Inflation rate \(= \frac{(39.98 - 36)}{36} \times \frac{100}{1}\)
   \[ = 11.1\% \]

2. \[ P35\, 890 = P25\, 000(1 + i)^3 \]
   \[ \therefore i \approx 0.07499 \]
   \[ \approx 7.5\% \]

3. \[ A = 7.89 \times 1.043 \]
   \[ = P8.23 \]

4. Inflation rate \(= \frac{-P29.10}{P289.45} \times \frac{100}{1}\)
   \[ = -10.1\% \]

**Note:** Point out that negative inflation means that the price of consumer goods decreases. Although it is unlikely to happen in present times, it is not impossible.

5. \[ A = P135(1 + 0.073)^2 \]
   \[ = P155.43 \]

**Revision Exercise**

(Student’s Book page 71)

1. The National Bank: \[ A = P12\, 000(1 + 0.105)^5 \]
   \[ = P19\, 769 \]

   Invest Bank: \[ A = P12\, 000(1 + \frac{0.102}{2})^{10} \]
   \[ = P19\, 734 \]

   The National Bank’s deal is better by P35, which is insignificant over a 5-year period.

2. \[ P78\, 600 = P49\, 000(1 + i)^3 \]
   \[ \therefore i \approx 0.1706 \]
   \[ \approx 17.1\% \]

3. a) \[ P1\, 320\, 000 = P682\, 000(1 + i)^5 \]
   \[ \therefore i \approx 0.1411 \]
   \[ \approx 14.1\% \]

   b) \[ P141\, 429 - P73\, 071 = P68\, 358 \text{ more VAT} \]

4. \[ P8\, 200 = P5\, 600(1 + \frac{i}{4})^{12} \]
   \[ \therefore i = 4 \times \left(\sqrt[12]{\frac{P8\, 200}{P5\, 600}} - 1\right) \]
   \[ \approx 0.1291 \]
   \[ \approx 12.9\% \]
5. a) \[ P5.50 \times 1.0337 = P6.90 \]
b) \[ P5.50 = P3.25(1 + i)^3 \]
\[ \therefore i \approx 0.1916 \]
\[ \approx 19.2\% \]

6. Simple interest: \[ A = 1(1 + 1000 \times 0.105) \]
\[ = P106 \]

Compound interest: \[ A = 1(1 + 0.105)^{1000} \]
\[ = P2.3029 \times 10^{43} \]

7. a) \[ \text{Interest} = P50,000 \times \frac{0.095}{4} \]
\[ = P1,187.50 \]
b) \[ A = P50,000(1 + \frac{0.095}{4})^{12} \]
\[ = P66,266.95 \]

8. Depreciated value = \[ P150,000(0.85)^5 \]
\[ = P66,555.80 \]
New car price = \[ P150,000(1.12)^5 \]
\[ = P264,351.25 \]
Patrick will need \[ P264,351.25 - P66,555.80 = P197,795.45 \].
Syllabus: General objective

This chapter deals with matrices and is an extension of the work covered in Form 2. In this chapter, students learn how to work with $4 \times 4$ matrices and some of the operations that are applicable to matrices.

Specific objectives

- Multiply matrices that comprise up to 4 columns and 4 rows.
- Investigate the properties of associativity, commutativity and identity of $2 \times 2$ matrices in arithmetic computations.

Ensure that students can manipulate matrices algebraically and use a calculator to perform calculations that involve matrices.

Answers

Support Activity 5.1

1. Monday \[\begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 0 \\ 3 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix}\]
2. Meat \[\begin{pmatrix} 42.00 \\ 7.50 \\ 8.50 \end{pmatrix}\]
3. No, they are not in the same order.

Extension Activity 5.1

1. $0 =$ no incident of influenza passing from one person to another
   $1 =$ one incident of influenza passing from one person to another
2. Column A: from whom person A caught influenza
   Column B: from whom person B caught influenza
   Column C: from whom person C caught influenza
   Column D: from whom person D caught influenza
Row A: to whom person A passed on influenza  
Row B: to whom person B passed on influenza  
Row C: to whom person C passed on influenza  
Row D: to whom person D passed on influenza  
3. The diagonal represents the values for the relationship of the same person,  
i.e. A to A, B to B, C to C and D to D.  
4. For example:

![Diagram showing relationships between A, B, C, and D]

**Exercise 5.1**  
*(Student’s Book page 78)*

1. A: $3 \times 1$ column matrix  
   B: $3 \times 3$ square matrix  
   C: $2 \times 2$ square matrix  
   D: $1 \times 4$ row matrix  
   E: $2 \times 1$ column matrix  
   F: $1 \times 2$ row matrix  
   G: $2 \times 3$ row matrix  
2. a) Equal  
   b) Not equal  
3. You can only add and subtract matrices of the same order.  
4. a) $\begin{pmatrix} 8 & 14 \\ 5 & 13 \end{pmatrix}$  
   b) $\begin{pmatrix} 4 & 9 \\ 10 & -10 \end{pmatrix}$  
   c) $(4 \ 14 \ 16)$  
   d) $\begin{pmatrix} 14 \\ -28 \\ -42 \\ -49 \end{pmatrix}$  
   e) $\begin{pmatrix} x & 4x \\ 8x & 6x \\ 3x & 2x \end{pmatrix}$  
   f) $\begin{pmatrix} 9 & 0 & 15 \\ 132 & 63 & -90 \\ -12 & 27 & -33 \end{pmatrix}$
Chapter 5: Matrices

Activity 5.1  
(Student’s Book page 81)

1. Product equals \( \begin{pmatrix} R_1 \times C_1 & R_1 \times C_2 \\ R_2 \times C_1 & R_2 \times C_2 \end{pmatrix} \) for \( 2 \times 2 \) matrices.

In other words, multiply each element in row 1 by each element of the same position in column 1, and so on.

**Note:** Ensure that students understand how to add and subtract matrices and why it is important to know this skill. You can introduce the use of a calculator here, but ensure that students first understand the mechanism of multiplication.

2. DE: \((3 \times 0) + (6 \times -1)\)  
DF: \((1 \times 2) + (5 \times 5)\)  
KF: \((3 \times 1) + (1 \times 3); (3 \times 2) + (1 \times 5)\)  
KF: \((1 \times 2) + (3 \times 5); (-2 \times 1) + (1 \times 3); (-2 \times -1) + (1 \times 2)\)

3. The first row and the second column give the order. A \(2 \times 2\) times a \(2 \times 2\) yields a \(2 \times 2\) matrix; a \(4 \times 2\) times a \(2 \times 3\) yields a \(4 \times 3\) matrix.

Activity 5.2  
(Student’s Book page 83)

1. a) Yes; \(2 \times 4\)  
b) Yes; \(3 \times 6\)  
c) Yes; \(1 \times 1\)  
d) No  
e) Yes; \(1 \times 3\)  
f) No  
g) No  
h) No  
i) Yes; \(10 \times 12\)  
j) No  
2. a) Any two matrices where the number of columns in matrix 1 is equal to the number of rows in matrix 2.  
b) Any two matrices where the condition in question 2a) is not met.

Exercise 5.2  
(Student’s Book page 84)

1. No product is possible.

2. \(1 \times 2; (23 \ 15)\)
3. \(3 \times 4; \begin{pmatrix} 88 & -68 & -45 & 16 \\ 225 & -130 & -50 & 100 \\ 59 & -54 & -76 & 4 \end{pmatrix}\)

4. \(2 \times 3; \begin{pmatrix} -19 & -20 & -26 \\ -11 & -33 & 81 \end{pmatrix}\)

5. \(1 \times 3; \begin{pmatrix} 120 & 132 & -36 \end{pmatrix}\)

6. No product is possible.

7. \(2 \times 2; \begin{pmatrix} 3 & 5 \\ -2 & 3 \end{pmatrix}\)

**Note:** Ensure that students are aware of the result when multiplying the identity matrix. We shall deal with this concept formally later in the chapter.

8. \(1 \times 4; \begin{pmatrix} 14 & -11 & 14 & 3 \end{pmatrix}\)

9. \(4 \times 4; \begin{pmatrix} 1 & 8 & 5 & 4 \\ 3 & 12 & 7 & 5 \\ 5 & 9 & -7 & 3 \\ 2 & 5 & -4 & 2 \end{pmatrix}\)

## Activity 5.3
*(Student’s Book page 85)*

1. \(B \times A\)
2. \(\begin{pmatrix} 3 & 300 \\ 380 \\ 220 \end{pmatrix}\)

3. The total number of Economy seats in their fleet is 3 300, the total number of Business seats is 380 and the total number of First class seats is 220.

## Exercise 5.3
*(Student’s Book page 86)*

1. Wednesday: P206.50  
   Thursday: P254.50  
   Friday: P303.00  
   Saturday: P442.00  
   Total: P1 206.00  

2. a) \((550 \quad 180)\)  
   b) Store A: P8 200  
      Store B: P9 840  
      Store C: P6 380  
      Total: P24 420
Extension Activity 5.2  (Student’s Book page 86)

1. \[
\begin{pmatrix}
4 & 500 \\
3 & 800
\end{pmatrix}
\]
2. \[
\begin{pmatrix}
3 & 840 \\
4 & 460
\end{pmatrix}
\]
3. 3 840 is the population of the inner-city and 4 460 is the population of the urban area.
4. Each column represents 100% population of either inner-city or urban area, and hence proportions (or ratios) add up to 1.

Project  (Student’s Book page 87)

1. Type
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
</table>
   Tomato | 3 | 2 | 0 |
   Cheese | 2 | 0 | 3 |
   Ham    | 0 | 1 | 1 |
2. \[
\begin{pmatrix}
15 & 30 \\
10 & 20 \\
15 & 20
\end{pmatrix}
\]
3. \[
\begin{pmatrix}
55 & 130 \\
75 & 120 \\
23 & 40
\end{pmatrix}
\]

Investigation  (Student’s Book page 88)

Matrix addition is commutative.
Matrix subtraction is not commutative.
Matrix multiplication is not commutative.

Investigation  (Student’s Book page 89)

1. True
2. Not True
Matrix addition is associative and matrix subtraction is not associative.

Exercise 5.4  (Student’s Book page 92)

1. a) Commutative Law for addition
   b) Associative Law for addition
   c) Associative Law for multiplication
2. a) False
   b) False
c) True
d) True
e) True
f) False
g) True
h) False

3. a) \( T - V = \begin{pmatrix} -2 & 2 \\ 1 & 2 \end{pmatrix} \)
    \[ V - T = \begin{pmatrix} 2 & -2 \\ 1 & -2 \end{pmatrix} \]
    \[ T - V \neq V - T \]

b) After removing the brackets, you can see that \( V - T - H \neq V - T + H \).

f) \( VH = \begin{pmatrix} 13 & 18 \\ -7 & 10 \end{pmatrix} \)
    \[ HV = \begin{pmatrix} 4 & -1 \\ 0 & -1 \end{pmatrix} \]
    \[ VH \neq HV \]

h) \( V + I_2 = \begin{pmatrix} 4 & -1 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \)
    \[ = \begin{pmatrix} 5 & 1 \\ -2 & 0 \end{pmatrix} \]
    \[ \neq V \]

**Homework Exercise** *(Student’s Book page 92)*

1. For real numbers \( a \) and \( b \), \( a \times b = 0 \) means either \( a = 0 \) or \( b = 0 \).

\[ AB = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \]
   A matrix is made of different entries.

2. For real numbers, if \( da = dc \), then \( a = c \).

\[ DA = \begin{pmatrix} 8 & 0 \\ 4 & 0 \end{pmatrix} \quad DC = \begin{pmatrix} 8 & 0 \\ 4 & 0 \end{pmatrix} \]

Multiplying matrix A and C give the same results when matrix A is NOT equal to matrix C.

**Revision Exercise** *(Student’s Book page 93)*

1. a) \( \begin{pmatrix} 7 & 36 \\ 10 & 35 \end{pmatrix} \)

b) \( \begin{pmatrix} -2 & -4 \\ 22 & -11 \end{pmatrix} \)

c) \( \begin{pmatrix} 70 & 20 \\ 18 & 16 \end{pmatrix} \)
Chapter 5: Matrices

d) \[
\begin{pmatrix}
2 & 4 & 6 \\
18 & 12 & 10 \\
16 & 8 & 20 \\
\end{pmatrix}
\]
e) Not possible

f) \[
\begin{pmatrix}
0 & 8 & -7 & -3 \\
5 & 16 & -3 & -1 \\
-2 & 3 & -15 & 1 \\
1 & 5 & 4 & -2 \\
\end{pmatrix}
\]

2. a) True
b) False
c) False
d) False

3. AC BC CA

4. When adding matrices you can change the order of the matrices.

5–7. Check own answers.
Assessment 1

Chapters 1–5

Question 1
1.1 \( T_n = -6 + 3(n - 1) = 3n - 9 \) ✓✓✓
1.2 \( T_{53} = 150 \) ✓✓
1.3 \( T_6 = -6 + 3(6 + 1) = 9 \); \(-6 + (-3) + 0 + 3 + 6 + 9 = 9 \) ✓✓
1.4 \( n = 150 \) ✓✓✓

[11]

Question 2
2.1 a) \((2a^2)^3\) or \(8a^6\) ✓✓✓
   b) \(8a^6\) ✓✓✓
   c) \((2x^3)^2\) or \(4x^6\) ✓✓✓✓
2.2 a) \(1.23 \times 10^{-6}\) ✓✓
   b) \(5.6 \times 10^6\) ✓✓
2.3 \(1.275 \times 10^{-11}\) ✓✓
2.4 a) \(\frac{71}{99}\) ✓✓✓
   b) It is rational. ✓

[21]

Question 3
3.1 a) P21 735.51 ✓✓✓✓
   b) P20 282.30 ✓✓
3.2 P20 282.30 ✓✓✓✓✓
3.3 P5.73 ✓✓✓

[14]

Question 4
4.1 a) X: \(3 \times 3\); Y: \(3 \times 3\) ✓
   b) i) \[
\begin{pmatrix}
1 & -2 & 5 \\
2 & 10 & 6 \\
1 & 6 & 2
\end{pmatrix}
\]
   ✓✓✓✓
   ii) \[
\begin{pmatrix}
7 & 0 & 7 \\
-1 & 10 & 6 \\
3 & 8 & 7
\end{pmatrix}
\]
   ✓✓✓
4.2 a) \(A + B = B + A = \begin{pmatrix} 2 & -1 \\ 4 & -1 \end{pmatrix}\) ✓✓
   b) \(AB \neq BA \Rightarrow \begin{pmatrix} 1 & -3 \\ -2 & -8 \end{pmatrix} \neq \begin{pmatrix} -6 & 4 \\ 5 & -1 \end{pmatrix}\) ✓✓
   c) \(A \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\) ✓
   \(B \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\) ✓

[14]

Total: 60
Syllabus: General objective

This chapter deals with the properties of triangles.

Specific objectives

- Derive the Midpoint theorem by investigation.
- Derive the converse of the Midpoint theorem by investigation.
- Solve problems involving the Midpoint theorem.
- Solve problems involving the converse of the Midpoint theorem.
- Calculate unknown sides and angles using properties of congruent triangles.
- Calculate unknown sides using properties of similar triangles.
- Solve problems involving properties of congruent triangles and similar triangles.

Properties of triangles are of great importance in all aspects of mathematics and mathematics-based disciplines, particularly where geometry is concerned. Many geometrical problems involve triangles, and knowledge of properties of triangles can be used to solve a variety of problems, both in further mathematics courses and real-life situations or professions such as architecture.

Answers

Activity 6.1  (Student’s Book page 101)

1–3. In triangle MNO below, A is the midpoint of side MN, line AR is parallel to side NO and it cuts side MO at B so that is MB = BO.
4. A parallelogram, AR||NO and RO||MN.
5. AN = AM (constructed A as Midpoint of MN)
   AN = RO (opposite sides of parallelogram)
6. \( \triangle \text{MAB} \) and \( \triangle \text{BOR} \) are the same shape and size (AM = RO; opp \( \angle \)s at B;
   \( \triangle \text{MAB} = \triangle \text{BRO} \))
7. MB and BO are equal.
8. B
9. The converse of the midpoint theorem

**Support Activity 6.1** *(Student’s Book page 102)*

KL = 6.4 cm

**Exercise 6.1** *(Student’s Book page 103)*

1. 66 mm
2. a)

```
    M
   /|
  /  |
A---B---T
     |
   /|
  /  |
    R
```

b) \( \triangle \text{MBA} \) (MB = MA) and \( \triangle \text{MTR} \) (MT = MR)
3. GK||LO (G, K midpoints, so GK||LM)
   KO||GL (K, O midpoints, so KO||LP)
   ∴ LGKO is a parallelogram (2 pairs of opposite sides parallel)
Extension Activity 6.1  
(Student’s Book page 103)

1. 23 cm
2. a) 39.8 cm  
   b) 6.64 cm  
   c) 229.1 cm²

Support Activity 6.2  
(Student’s Book page 104)

1. 

```
```

2. YG = 5.2 cm (Midpoint theorem)
3. GH = 5.2 + 5.2 = 10.4 cm

Exercise 6.2  
(Student’s Book page 104)

1. The straight line drawn through the midpoint of one side of a triangle parallel to another bisects the third side.

2. 

```
```

3. She needs to pull a line parallel to the first floor until she meets the third side of the roof. That point will be the bisector of that side.
4. a) Since D is the midpoint of CF and DE is parallel to FG, then according to the converse of the midpoint theorem, E is the midpoint of CG and bisects CG.  
   b) B divides CA into two equal parts. Therefore CB is \( \frac{1}{2} AC = 3.1 \) cm.
5. a) Angle ADE = 80º (Corresponding angles)
   b) Angle FEC = 40º (Corresponding angles)
   c) EF = 5 cm. D is the midpoint because E is the midpoint of AC and ED is parallel to BC. Hence DB = EF because of a parallelogram.

### Extension Activity 6.2
(Student’s Book page 106)

Scale factor of length in the triangles is half. Therefore the scale factor of the areas is one quarter.

\[ \therefore \text{Area } \triangle BAC = 20 \text{ cm}^2 \]

\[ \therefore \text{Area } BKMC = 20 \text{ cm}^2 - 5 \text{ cm}^2 = 15 \text{ cm}^2 \]

### Activity 6.2
(Student’s Book page 106)

SSS: Enough
AAA: Not enough
SAS: Enough
SSA: Not enough
SAA: Enough
RSH: Enough

### Support Activity 6.3
(Student’s Book page 108)

1. a) FJ = 7.5 cm
   b) QJ = 5.2 cm
   c) RQ = 6.8 cm
2. \( \angle FJQ \)
3. The formed alternating angles are equal.

### Exercise 6.3
(Student’s Book page 109)

1. PQM and PSM (RSH)
   PQR and PSR (SSS)
   QMR and SMR (RSH)
2. MO = 10 cm, \( \angle N = 60^\circ \)
3. a) XOY and WOZ are congruent.
   b) They are congruent because of the SAS rule.
   c) XY = WZ because \( \triangle XOY \equiv \triangle WOZ \).
Support Activity 6.4  (Student’s Book page 110)

5. Ratios are equal

Homework Activity  (Student's Book page 112)

1. AAA (3 pairs of corresponding angles are equal)

Exercise 6.4  (Student’s Book page 113)

1. a) $\triangle EMP$ and $\triangle JFK$
   
   b) $\frac{EM}{JF} = \frac{1}{3}$
   
   c) $\frac{EM}{JF} = \frac{1}{3}$
   
   d) i) $EP = 3$ cm
      
      ii) $FK = 12.9$ cm
   
   2. $\angle ACB = \triangle RCT$ (Vertically opposite angles)
      
      $\angle CAB = \angle CRT$ (Alternating angles and parallel lines)
      
      $\angle ABC = \angle RTC$ (Alternating angles and parallel lines)
      
      .: by the AAA property the triangles are similar
   
   3. b) i) 6 cm ii) 4.8 cm iii) 9 cm iv) 4 cm v) 5.6 cm
   
   4. a) Yes, AAA
      
      b) $\triangle ABR \parallel \triangle YXR$
      
      c) $\frac{AR}{RY} = \frac{16}{9}$
      
      d) $AB = \frac{1.3 \times 3.2}{1.8} = 2.3$ m

Exercise 6.5  (Student’s Book page 116)

1. a) $x = 7.2$ cm b) $p = 70^\circ$ and $y = 40^\circ$ c) $z = 6.3$ cm and $q = 40^\circ$

2. a) SAS
      
      b) $\frac{8}{4.8} = \frac{k + 4.2}{4.2} \therefore k = 2.8$ cm
      
      c) $\frac{g}{6} = \frac{8}{4.8} \therefore g = 10$ cm

3. a) Ratios: $\frac{3.4}{5.1} = \frac{2}{3}$ and $\frac{4.5}{6.75} = \frac{2}{3}$
      
      The two corresponding sides are equal and the angle included, so the triangles are similar.
      
      b) The scale factor of reduction is $\frac{2}{3}$.

4. a) $DE \parallel BF$ and $AD = DB$
      
      b) $DE = \frac{1}{2} BC$ (D and E are midpoints of AB and AC)
      
      .: DE bisects AC 
      
      .: $AE = EC$
      
      .: $2DE = BC$
Extension activity 6.3

1. \( EI = 7.2 \text{ cm} \)

2. If \( P \) is the midpoint of \( AB \), \( Q \) the midpoint of \( BC \) and \( R \) the midpoint of \( AC \), then:
   - In \( \triangle RQC \) and \( \triangle APR \): \( RC = AR \) (Midpoint theorem)
   - \( AP = RQ \) (Midpoint theorem)
   - \( PR = QC \) (Midpoint theorem)

   By SSS, the two triangles \( RQC \) and \( APR \) are congruent.
   
   The same reasoning works for \( \triangle PBQ \) with \( APR \) and/or \( RQC \).
   
   For \( \triangle QRP \) and \( \triangle PBQ \):
   - \( BQ = RP \) (Midpoint theorem)
   - \( BP = RQ \) (Midpoint theorem)
   - \( PQ \) is common

   Therefore by SSS the two triangles are congruent.

   It follows that all four triangles in \( \triangle ABC \) are congruent.

3. Ratio of areas \( PQR:PST:PVW \)

   (Remember: ratio of areas is the square of length of sides.)
   
   Ratio of \( PR:PT:PW = 2:4:6 = 1:2:3 \)
   
   Ratio of \( PQR:PST:PVW = 1:4:9 \)

Revision Exercise

1. a) \( ZY = 6.2 \text{ cm} \)
   b) \( XY = 9.6 \text{ cm} \)

2. a) \( PA = 5.9 \text{ cm} \)
   b) \( PH = 4.7 \text{ cm} \)
   c) \( \angle APH = 60^\circ \)
   d) \( \angle PHY = 95^\circ \)

3. a) \( \triangle A \) and \( \triangle C \) (SAS and ratios of corresponding sides are equal)
   b) \( \triangle D \) and \( \triangle F \) (SAS and ratios of corresponding sides are equal)
   c) All triangles are similar

4. a) \( p = 9.3 \text{ cm} \)
   b) \( k = 2.4 \text{ cm} \)
   c) \( m = 5.07 \text{ cm} \)

5. \( BD = 6 \text{ cm} \)

6. The triangles are similar because of AAA. Scale factor is \( \frac{3}{4} \).
Syllabus: General objective
This chapter deals with the Pythagorean theorem.

Specific objectives
- Derive the Pythagorean theorem by investigation.
- Calculate the unknown sides of a right-angled triangle using the Pythagorean theorem.
- Use the Pythagorean theorem to solve problems related to real-life situations.

The Pythagorean theorem is a very powerful and useful theorem that can be applied to calculate sizes and length (or distances) in right-angled triangular shapes and objects. It can be employed to minimise and maximize resources where necessary in various practical situations involving right-angled triangles.

Therefore, it is important to expose students to this theorem and its importance, since they always encounter situations in their life in which understanding of the theorem will be handy.

The trigonometric ratios are also very important, as they can be used to achieve accuracy in angle sizes and lengths of sides of right-angled triangular shapes and objects.

Answers

Investigation

1. a) ST
   b) 9 cm², 16 cm² and 25 cm²
   c) $25 = 9 + 16$; the area of the square on the longest sides is equal to the sum of the areas of the other squares.

2. a) 169
   b) 25 and 144
   c) $169 = 144 + 25$; again the area of the square on the largest side of the triangle equals the sum of the areas of the other squares.
3. | Diagram | Area of square A | Area of square B | Area of square C | Sum of areas of squares A and B |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>144</td>
<td>25</td>
<td>169</td>
<td>169</td>
</tr>
<tr>
<td>(b)</td>
<td>16</td>
<td>9</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>(c)</td>
<td>64</td>
<td>225</td>
<td>289</td>
<td>289</td>
</tr>
</tbody>
</table>

4. Students should have observed that in a right-angled triangle, the area of the square on the hypotenuse is always equal to the sum of the areas of the squares on the other two sides of the triangle.

Note: This is also true for semi-circles, equilateral triangles and numerous other equilateral shapes. Let the students investigate this further.

Support Activity 7.1 *(Student’s Book page 130)*
1. 15 cm
2. 12 cm
3. 12 cm
4. 4.9 cm

Exercise 7.1 *(Student’s Book page 131)*
1. $b = 28$ cm, $c = 7.5$ cm
2. $s = 2.5$ cm, $t = 1.4$ cm
3. $b = 4$, $a = 15$
4. $b = 4.9$ cm, $a = 10$ cm, $c = 14.9$ cm
5. 5 cm
6. AC = 12 cm

Extension Activity 7.1 *(Student’s Book page 132)*

![Diagram](image)

Homework Activity *(Student’s Book page 132)*
One side is 1 cm. The other side is the hypotenuse of the previous triangle.
Exercise 7.2  
(Student’s Book page 133)

1. No
2. Yes, $\hat{T} = 90^\circ$
3. Yes, $\hat{T} = 90^\circ$

Exercise 7.3  
(Student’s Book page 135)

1. 18 sec
2. Height = 15 m
3. $H = 3.9$ m
4. 21.8 km
5. 1.7 m
6. 12.1 m

Revision Exercise  
(Student’s Book page 137)

1. a) 4 cm  
   b) 2.3 cm  
   c) 12.46 cm  
2. $d = 14.48$ cm  
3. $AB = 2.5$ m  
4. 7.07 cm  
5. 17 cm  
6. $XY = 25.3$ m
Syllabus: General objective
This chapter deals with trigonometric ratios.

Specific objectives
- Identify opposite, adjacent and hypotenuse sides of a right-angled triangle with reference to the given angle.
- Calculate the three trigonometric ratios of sine, cosine and tangent in a right-angled triangle.
- Use the three trigonometric ratios and a calculator to calculate the unknown angle and side in a right-angled triangle.
- Use both the trigonometric ratios and the Pythagorean theorem to solve problems related to real-life situations.

Answers

Support Activity 8.1
(Student's Book page 141)

1. a)
Exercise 8.1  
(Student’s Book page 141)

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Hypotenuse</th>
<th>Opposite</th>
<th>Adjacent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>5 cm</td>
<td>3 cm</td>
<td>4 cm</td>
</tr>
<tr>
<td>(b)</td>
<td>13 cm</td>
<td>5 cm</td>
<td>12 cm</td>
</tr>
<tr>
<td>(c)</td>
<td>10 cm</td>
<td>6 cm</td>
<td>8 cm</td>
</tr>
<tr>
<td>(d)</td>
<td>$z$</td>
<td>$y$</td>
<td>$x$</td>
</tr>
<tr>
<td>(e)</td>
<td>$p$</td>
<td>$r$</td>
<td>$q$</td>
</tr>
</tbody>
</table>

Support Activity 8.2  
(Student’s Book page 144)

1. | Triangle | Hypotenuse | Opposite | Adjacent |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\sqrt{34}$</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>(b)</td>
<td>10</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

2. a) $\sin \theta = \frac{32}{\sqrt{34}}$ \quad $\cos \theta = \frac{5}{\sqrt{34}}$ \quad $\tan \theta = \frac{3}{5}$
   b) $\sin \theta = \frac{6}{10}$ \quad $\cos \theta = \frac{8}{10}$ \quad $\tan \theta = \frac{6}{8}$

Activity 8.1  
(Student’s Book page 144)

1. a) The length of hypotenuse is 13 cm.  
   The opposite side is 5 cm.  
   The adjacent side is 12 cm.
   b) The length of hypotenuse is 41 cm.  
   The opposite side is 9 cm.  
   The adjacent side is 40 cm.
   c) The length of hypotenuse is 5 cm.  
   The opposite side is 4 cm.  
   The adjacent side is 3 cm.

2. a) $\sin \alpha = \frac{9}{41}$; $\cos \alpha = \frac{12}{13}$; $\tan \alpha = \frac{5}{12}$
   b) $\sin \alpha = \frac{9}{41}$; $\cos \alpha = \frac{40}{41}$; $\tan \alpha = \frac{9}{40}$
   c) $\sin \alpha = \frac{4}{5}$; $\cos \alpha = \frac{3}{5}$; $\tan \alpha = \frac{4}{3}$
   d) $\sin \alpha = \frac{12}{13}$; $\cos \alpha = \frac{5}{13}$; $\tan \alpha = \frac{12}{5}$
   e) $\sin \alpha = \frac{7}{\sqrt{170}}$; $\cos \alpha = \frac{11}{\sqrt{170}}$; $\tan \alpha = \frac{7}{11}$
Exercise 8.2

(Student’s Book page 146)

1. a) \( \frac{AC}{AB} \cdot \frac{CB}{AB} \)
   b) \( \frac{FE}{HE'} \cdot \frac{HF}{HE} \)
   c) \( \frac{XY}{WY} \cdot \frac{WX}{WY'} \cdot \frac{XY}{WX} \)
   d) \( \frac{NT}{KT} \cdot \frac{KN}{KT} \)
   e) \( \frac{SU}{ST} \) or \( \frac{TU}{ST} \)

2 a) \( \sin \angle = \frac{3}{5}, \cos \angle = \frac{4}{5}, \tan \angle = \frac{3}{4} \)
   b) \( \sin \angle = \frac{12}{13}, \cos \angle = \frac{5}{13}, \tan \angle = \frac{12}{5} \)
   c) \( \sin \angle = \frac{1}{\sqrt{2}}, \cos \angle = \frac{1}{\sqrt{2}}, \tan \angle = 1 \)
   d) \( \sin \angle = \frac{4}{5}, \cos \angle = \frac{3}{5}, \tan \angle = \frac{4}{3} \)
   e) \( \sin \angle = \frac{15}{17}, \cos \angle = \frac{8}{17}, \tan \angle = \frac{15}{8} \)

3. There are alternative answers such as the following:
   a) \( BC = 10 \sin A, BC = 6 \cos B, BC = 6 \tan A \)
   b) \( EF = 13 \cos G, EF = 5 \tan G, EF = 13 \cos F \)
   c) \( KL = \frac{\sqrt{3}}{\sin k} \)

Exercise 8.3

(Student’s Book page 150)

1. a) 22.6°  b) 29°  c) 60°  d) 20.9°  e) 61°  f) 77°
2. a) 1.33  b) 0.74  c) 0.866  d) 1  e) 0.60  f) 0.21
3. a) 6.69  b) 4.62  c) 2.65  d) 11.2  e) 5.36  f) 12.8
   g) 6.39  h) 6  i) 11.7

Extension Activity 8.1

(Student’s Book page 151)

\( \angle BAD = 97.6° \)

Activity 8.4

(Student’s Book page 151)

1.

2. Yes

3. 30°

Exercise 8.4

(Student’s Book page 152)

1. 4.3 cm  2. 4.3 m  3. a) 4.56 m  b) 37.9°

Extension Activity 8.2

(Student’s Book page 153)

1. \( \cos 60° = \frac{BC}{8} \)
   \( BC = 8 \cos 60° \)
   \( = 4 \)
   \( BD = \sqrt{4^2 + 4^2} \)
   \( = 5.66 \text{ m} \)
2. \( \tan \theta = \frac{1}{5} \), where \( \theta \) is the angle that the road makes with the horizontal.
\[ \therefore \theta = 11.3^\circ \]

**Revision Exercise**

1. a) \( \angle A = \cos^{-1} \left( \frac{4}{7} \right) = 55.2^\circ \), \( \angle C = \sin^{-1} \left( \frac{4}{7} \right) = 34.8^\circ \), and \( BC = 5.74 \text{ cm} \)
   b) \( \angle A = 180 - (90 + 70) = 20^\circ \), \( AB = 4.5 \tan 70 = 12.4 \text{ cm} \), and
   \[ AC = \sqrt{12.4^2 + 4.5^2} = 13.2 \text{ cm} \]
2. \( \sin \theta = \frac{1.5}{2.5} \), where \( \theta \) is the angle made by the door \( AB \) and the roof
   \[ \therefore \theta = \sin^{-1} \left( \frac{1.5}{2.5} \right) \]
   \[ \therefore \theta = 36.9^\circ \]
3. \( \tan \beta = \frac{15}{8} \), where \( \beta \) is the angle at \( F \)
   \[ \therefore \beta = \tan^{-1} \left( \frac{15}{8} \right) \]
   \[ \therefore \beta = 61.9^\circ \]
4. a) \( \sin 26^\circ = \frac{PS}{4} \), so \( PS = 4 \sin 26^\circ = 1.75 \text{ m} \)
   b) \( \cos 26^\circ = \frac{AP}{4} \), so \( AP = 4 \cos 26^\circ = 3.6 \text{ m} \)
   \[ AB = 7 = AP + PQ + QB \]
   \[ \therefore 7 = AP + PS + QB, \text{ since } PQ = PS \]
   \[ \therefore 7 = 4 \cos 26^\circ + 4 \sin 26^\circ + QB \]
   \[ \therefore QB = 1.65 \text{ m} \]
   c) \( RB^2 = QB^2 + PS^2 \)
   \[ \therefore RB = \sqrt{QB^2 + PS^2} \]
   \[ = \sqrt{5.785} \]
   \[ = 2.41 \text{ m} \]
   d) \( QR = \frac{4 \sin 26^\circ}{RB} \)
   \[ = 46.6^\circ \]
5. \( \tan VXY = \frac{4}{25} \)
   \[ \therefore \angle VXY = 9.09^\circ \]
6. a) \( \angle AOB = \frac{360}{5} = 72^\circ \) and \( \angle OAB = \frac{180 - 72}{\tan 36} = 54^\circ \)
   b) Area \( \triangle AOB = \frac{1}{2} \times 20 \times \frac{10}{\tan 36} \)
   \[ = 137.6 \text{ cm}^2 \]
   c) Area pentagon = \( 137.6 \times 5 \)
   \[ = 688.2 \text{ cm}^2 \]
Syllabus: General objective

This chapter deals with the use of coordinates in a Cartesian plane.

Specific objectives

- Calculate the distance between two points.
- Calculate the coordinates of the midpoint of a line segment given the coordinates of its endpoints.
- Calculate the coordinates of the endpoint of a line segment given the coordinates of the midpoint and one end point.
- Solve problems involving the applications of distance between two points and midpoint of the two endpoints of a line segment.

Answers

**Activity 9.1**  
(Student’s Book page 158)

1. A(1;2), B(6;8)
4. C(6; 2)
5. AC = 6 - 1 = 5 units; BC 8 - 2 = 6 units
6. AB = $\sqrt{5^2 + 6^2} = 7.8$ units
7. AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Exercise 9.1**  
(Student’s Book page 160)

1. a) Distance = $\sqrt{(1 - 9)^2 + (-4 - 14)^2}$
   = $\sqrt{(-8)^2 + (-18)^2}$
   = $\sqrt{64 + 324}$
   = 19.7 units
b) Distance = 10 units
c) Distance = 5.7 units
d) Distance = 9.06 units
2. a) AB = 4 units; AC = $\sqrt{29}$ units
   BC = $\sqrt{29}$ units → Isosceles
b) \( AB = 17.1 \) units; \( AC = 12 \) units; 
   \( BC = 17.1 \) units  \( \Rightarrow \) Isosceles 

c) \( AB = 6.3 \) units; \( AC = 8.5 \) units  
   \( BC = 10.4 \) units  \( \Rightarrow \) Not isosceles 

d) \( AB = 4.1 \) units; \( AC = 4.1 \) units; 
   \( BC = 2 \) units  \( \Rightarrow \) Isosceles 

**Extension Activity 9.1**  
(Student’s Book page 161) 

1. \( AD = 3 \) units; \( DB = 4 \) units 
2. \( 3 \) units in \( y \)-direction and \( 4 \) units in \( x \)-direction. 
3. \( AB = 5 \) units 
4. \( AB = \sqrt{(0 - 4)^2 + (3 - 0)^2 + (0 - 0)^2} = 5 \) units 
5. \( E \) is in the \( y-z \)-plane which means zero units in the \( x \)-direction, but \( 3 \) units in the \( y \)- and \( 2 \) units in the \( z \)-direction. 
6. \( AE = \sqrt{29} = 5.4 \) units 
7. \( AE = \sqrt{AB^2 + BE^2} = \sqrt{5^2 + 2^2} = 5.4 \) units 

**Activity 9.2**  
(Student’s Book page 162) 

1.–3. 

4. \( x = \frac{2 + 8}{2} = 5; \ y = \frac{3 + 7}{2} = 5, \) so \( c = (5;5) \) 
5. Depends on students’ drawing. 
6. \( x = \frac{x_1 + x_2}{2}, \ y = \frac{y_1 + y_2}{2} \) 
7. Students’ own answers. 

**Exercise 9.2**  
(Student’s Book page 162) 

1. a) \((5.5;5.5)\) 
   
   b) \((4;3.2)\)
c) \((-0.5;3)\)
d) \((-1;0.5)\)

2. a) 20 units
   b) 62.8 units
   c) \((8;9)\)

### Support Activity 9.1

(Student’s Book page 163)

1. \(\frac{-4 + x}{2} = 0; \therefore x = 4\)
   \(\frac{7 + y}{2} = 3; \therefore y = -1\)

2. \(\frac{-8 + x}{2} = -3; \therefore x = 2\)
   \(\frac{-6 + y}{2} = 4; \therefore y = 14\)

### Exercise 9.3

(Student’s Book page 164)

1. a) \((4;-1)\)
   b) \((2;14)\)
   c) \((-8;2)\)
   d) \((-3;-15)\)
   
2. b) C(3;8)
   c) D(3;4)

### Extension Activity 9.2

(Student’s Book page 164)

1. \((1;4); (1;0)\) and \((5;0)\)

2. Gradient AB \(= \frac{11 - 7}{4 - 2} = 2\)
   Gradient CB \(= \frac{11 - 1}{4 + 1} = 2\)
   So A, B and C are in a straight line.

3. Gradient AB \(= -\frac{4}{3}\)
   Gradient of \(\perp\) line is \(\frac{3}{4}\) (product of the gradients of perpendicular lines is \(-1\)).
   Line goes through the midpoint of AB which is \((1;2)\).
   Therefore: \(y - y_1 = m(x - x_1)\)
   \[\therefore y - 2 = \frac{3}{4}(x - 1)\]
   \[\therefore y = \frac{3}{4}x + \frac{5}{4}\]
Activity 9.3  
(Student’s Book page 166)

1. 

\[ B(8;4) \]

\[ A(-2;-6) \]

\[ M(3;-1) \]

\[ \theta = 135^\circ \]

2. Inclination = \( \tan \theta = \frac{4 - (-6)}{8 - (-2)} = 1 \)

\[ \theta = 45^\circ \]

3. 
   a) Strut supports at (3; -1)
   b) 135° with the ‘positive x-axis’
   c) Girder mass = \( 6 \times \sqrt{200} = 84.9 \) kg

Exercise 9.4  
(Student’s Book page 166)

1. Diameter = 13 units; Radius = 6.5 units
2. 
   a) \( AB = 2.8 \) units
   b) \( M(1;4) \)
   c) \( AC = 7.2 \) units; \( BC = 7.2 \) units.

So, \( ABC \) is isosceles.

   d) No, \( 7.2^2 \neq 7.2^2 + 2.8^2 \)
   e) i) Perimeter = 17.2 units
   ii) \( MC = \sqrt{50} = 7.1 \) units
   iii) \( MC \) is an altitude since

\[
m_{MC} \times m_{AB} = -1
\]

Area \( \triangle ABC = \frac{1}{2} \times AB \times MC \)

\[
= \frac{1}{2} \times 2.8 \times 7.1
\]

\[
= 9.9 \text{ units}^2
\]

3. 
   a) \( M(0.5;6) \)
   b) \( P \) is halfway between \( A \) and \( M \).

So, \( P(-2.75;4) \)

4. \( AP = 5.8 \) units, so \( A \) is not on the circle.

\( BP = 5 \) units, so \( B \) is on the circle.
Revision Exercise

(Student’s Book page 168)

1. Midpoint \[ \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right) \]
   Distance \[ \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

2. a) Length = 19.7 units
   Midpoint (5;5)
 b) Length = 10 units
   Midpoint (−4;−3)
 c) Length = 5.7 units
   Midpoint (1;1)
 d) Length = 9.1 units
   Midpoint (2.5;−4.5)

3. a) Not isosceles
 b) Isosceles
 c) Isosceles
 d) Isosceles

4. Radius = 5 units

5. a) M(2;1); N(3;9)
 b) MN = 2.236 units
   BC = 4.472 units
 c) Midpoint theorem as BC = 2MN
Syllabus: General objective

This chapter deals with the practical applications of plans, elevations and bearings.

Specific objectives

• Draw the front, side and plan elevations of the same shape and house.
• Find a point’s position using direction/bearings, distances and/or angles in a given journey.
• Use the Pythagorean theorem and trigonometric ratios to solve problems involving angle of elevation, angle of depression and bearings.

Answers

Exercise 10.1  
(Student’s Book page 173)

1. a) 

b) 

c)
2. a) 

b) 

3. a) 

b) 

c) 

d)
Activity 10.3  
(Student’s Book page 176)

2. a) Due North (000°)
    b) 045°
    c) 108.4°
    d) 206.6°

Activity 10.4  
(Student’s Book page 176)

Remember to always draw north lines parallel to each other.

The bearing of the village health clinic from Sadi's home measures to be 080°. The bearing of the direct route from the clinic = 360° – 100°, using the property of the sum of interior angles formed along a transversal line cutting parallel lines and the area around a point.

So, the bearing of the direct route = 260°.
Exercise 10.2  
(\textit{Student’s Book page 177})

2. a) 050°  
   b) 080°  
   c) 340°  
   d) 220°  
   e) 050°  
   f) 330°

3. a) \[ \begin{array}{c}
\text{C} \\
\text{55°} \\
\text{75°} \\
\text{B} \\
\text{50°} \\
\text{A}
\end{array} \]

   b) i) 050°  
      ii) 305°  
      iii) 230° (180° + 50°)

Homework Activity  
(\textit{Student’s Book page 178})

1. 270°  
2. 180°  
3. 000° (or 360°)

Extension Activity 10.1  
(\textit{Student’s Book page 179})

1. Let the bearing from B to A be \( x \).
   Thus the bearing from A to B would be 180° + \( x \)
   \[ \therefore 180° + x = 4x \]
   \[ x = 60° \]
   Bearing from B to A is 60° and the bearing from A to B is 180° + 60° = 240°

2. 180° + \( x \) = 7x
   \[ x = 30° \]
   B \( \rightarrow \) A is 30° and A \( \rightarrow \) B is 210°

Activity 10.5  
(\textit{Student’s Book page 180})

Elevation: Line of sight is at an angle upwards from the horizontal (look up from the horizontal).

Depression: Line of sight is at an angle downwards from the horizontal (cast your eyes downward from the horizontal).
Activity 10.6

1. [Diagram]

2. 48°
3. 222°
4. 13.5 km
5. \[
sin 48° = \frac{10}{13.5}
\]
   \[
   0.74 = 0.74 \text{ (confirmed)}
   \]

Exercise 10.3

1. a) 15 km
   b) 56.3°
   c) 326.3°
2. 20.6°
3. a) angle of depression
   b) 2.87°
4. 23.4°

Extension Activity 10.2

\[
AC + CB = \frac{210}{\cos 54°} + \frac{210}{\sin 75°} = 574.7 \text{ km}
\]
Revision Exercise

1. a) 

   ![Diagram](image1)
   
   plan  front  side

   ![Diagram](image2)
   
   plan  front  side

   ![Diagram](image3)
   
   plan  front  side

b) 

   ![Diagram](image4)
   
   plan  front  side

   ![Diagram](image5)
   
   plan  front  side

   ![Diagram](image6)
   
   plan  front  side

2. 

   ![Diagram](image7)

3. a) 

   ![Diagram](image8)
   
   plan  front  side

   ![Diagram](image9)
   
   plan  front  side

   ![Diagram](image10)
   
   plan  front  side

   (Student’s Book page 184)
5. b) i) 60°  
   ii) 240°  
   iii) 215°
6. $h = 150 \tan 35^\circ = 105 \text{ m}$
7. Distance = 37.3 metres
8. $\tan 37^\circ = \frac{1.7}{x}; x = 2.26 \text{ m}$
   Blind distance = $x - 0.85 = 1.4 \text{ m}$
9. a) $a = 60 \cos 30^\circ \approx 52 \text{ m}$
    b) $b = 60 \cos 20^\circ \approx 56.4 \text{ m}$
    c) $b - a = 4.4 \text{ m}$
Syllabus: General objective

This chapter deals with understanding and using combined transformations.

Specific objectives
• Draw combined transformations up to three different types.
• Identify and describe fully combined transformations involving up to three different types.
• Solve problems involving combined transformations.

Answers

Activity 11.1  (Student’s Book page 188)

1. a) Flip in a mirror line.
   b) Shift or slide through a point (usually (0;0)) over a certain distance, i.e. by a vector \( \begin{pmatrix} x \\ y \end{pmatrix} \).
   c) Rotated through a certain angle (usually 90° or 180°) relative to a reference point.
   d) A shape is made bigger or smaller by a constant factor (scale factor).

Exercise 11.1  (Student’s Book page 191)

1. a) Translation by vector \( \begin{pmatrix} 4 \\ 6 \end{pmatrix} \)
   b) Reflection in \( y = x \)
   c) Reflection in \( y = 2 \)
   d) Reflection in \( y = 0 \)
   e) Rotation about centre (0,0) through 90°

2. a) Image F: (-2.5;2.5), (-1;2.5) and (-1;1.5)

   Note: an enlargement of \( \frac{1}{2} \) is actually a reduction. To prevent confusion amongst students, it is safer to talk of a reduction by a factor \( \frac{1}{2} \), so just point this out.

   b) Image G: (-5;2), (-3;2) and (-5;5)
Exercise 11.2
(Student’s Book page 193)

1. Image triangle has vertices (1;6), (4;6) and (1;5) 
2. Image triangle has vertices (2;1), (3;–2) and (3;1) 
3. Image triangle has vertices (5;3), (5;4) and (8;3) 
4. Image triangle has vertices (4;1), (4;2.5) and (8.5;1) 

Exercise 11.3
(Student’s Book page 195)

1. a) Image triangle has vertices (–5;–5), (–3;–5) and (–5;–8). 
b) Rotation about centre (2;–3) through 180˚ followed by translation by vector \[ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \] or a rotation about (0;0) through 180° followed by a translation \[ \begin{pmatrix} 4 \\ –6 \end{pmatrix} \]. 
c) Rotation through –90˚ about centre (0;0) followed by reflection in \( x = 3 \) and then translation by vector \[ \begin{pmatrix} 2 \\ 0 \end{pmatrix} \]. 
d) Image triangle has vertices (–2;–4), (–2;–10) and (2;–4).
2. a) H: (5;–2); (5;–7); (2;–7) 
b) K: (7;–2); (7;–5); (2;–5) 
c) Reflection in \( y = 0 \) (x-axis) 
3. b) Enlargement, about centre (17;0) with scale factor 1.5.

Extension Activity 11.1
(Student’s Book page 196)

1. 

\[
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

as this is the identity matrix for multiplication.
2. \((4;1)\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = (-4; -1) (A_2), \quad (4;1)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (1;4) (A_3), \quad (4;1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (4;1)
\]
3. The matrix \(\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\), as shown by point \( A_2 \).
4. \(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\), as this is the identity matrix for multiplication.
5. The matrix \(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\)
6. \(\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\)
**Activity 11.4**
(Student’s Book page 197)

1. a) Reflection $x = 0$ or $y$-axis and translation $\left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right)$
   
   b) $C'(0; -4)$

2. Reflection in the $y$-axis or the line $x = 0$

**Exercise 11.4**
(Student’s Book page 197)

1. a) Translation vector $\left( \begin{array}{c} -5 \\ 3 \end{array} \right)$.
   
   b) B and C, reflection in $y = x$
   
   c) Triangle with vertices $(-0.5; -1), (0.5; -1)$ and $(-0.5; -2.5)$

2. Reflection in $y = x$ and translation by vector $\left( \begin{array}{c} 3 \\ -2 \end{array} \right)$.

3. A is point $(-2; -5)$.

4. Rotation about the origin through $180^\circ$.

5. Reflection in the $y$-axis, rotation about $(2; 2)$ through $90^\circ$ clockwise, reflection in the $x$-axis, enlargement about the origin with scale factor $\frac{1}{2}$.

**Revision Exercise**
(Student’s Book page 202)

1. c) $\triangle EFG$: $(-2; -4), (-1; 0)$ and $(3; -1)$

2. c) Rotation about the origin through $90^\circ$ anticlockwise.

3. Reflection in $y = x$.
   
   Enlargement by a scale factor 2 through the point $(-7; 1)$.

4. Rotation by $-90^\circ$ about $(0, 0)$
   
   (Rule: $(x; y) \rightarrow (y; -x)$)

5. Enlargement with scale factor $\frac{1}{2}$ and centre $(12; 0)$.

6. a) $P'Q'R'$: $(-2; -1), (-2; 1), (0; 1)$
   
   c) The original object was translated by the vector $\left( \begin{array}{c} -6 \\ -6 \end{array} \right)$ to give the final image.

**Assessment 2**
 Chapters 6 – 11

**Question 1**

1.1 a) $NZ = 60\checkmark$ mm (N midpoint since $MN\parallel ZY)$✓✓

   b) $MN = \frac{1}{2}ZY = 40$ mm ✓ ✓
   
   ML = 40 + 24 = 64 mm✓

   c) $YK = 2ML✓ = 128$ mm✓

1.2 a) $(2x - 3)^2 = x^2 + 9✓✓$

   b) $4x^2 - 12x + 9 = x^2 + 9✓$

   \[4x^2 - x^2 - 12x = 9 - 9✓\]
\[ \therefore 3x^2 - 12x = 0 \checkmark \]
\[ \therefore 3x(x - 4) = 0 \checkmark \]
\[ \therefore x = 0 \text{ or } x = 4 \therefore x = 4 \text{ units } \checkmark \]

1.3 a) \[ AC = \sqrt{40^2 + 40^2} \checkmark = 56.56 \text{ mm} \checkmark \]
b) Area = 3 200 mm\(^2\)

**Question 2**

2.1 \[ \tan 27^\circ = \frac{h}{150} \checkmark \therefore h = 150 \tan 27^\circ \checkmark = 76.4 \text{ m} \checkmark \]

2.2 a) \[ BE = 12 \tan 58.6^\circ \checkmark = 19.7 \text{ m} \checkmark \]
b) \[ AE = 12 \tan 63^\circ \checkmark = 23.6 \text{ m} \checkmark \]
c) \[ AB = AE - BE = 3.9 \text{ m} \checkmark \]

**Question 3**

3.1 \[ x = -1 \checkmark\checkmark \]
\[ y = -2 \checkmark\checkmark \]

3.2 a) Perimeter = \[ XY + XZ + YZ \checkmark \]
\[ = 10 \checkmark + \sqrt{34} \checkmark + \sqrt{146} \checkmark \]
\[ = 27.9 \text{ units} \checkmark\checkmark \]

b) 2.92 units \checkmark

Midpoints: \((-1;5) \checkmark\checkmark \) and \((\frac{1}{2};\frac{5}{2}) \checkmark\checkmark \)

**Question 4**

4. Plan:

Front: \[ \checkmark \]
Side: \[ \checkmark \checkmark \]

**Question 5**

5.1 – 5.4

[Graph of Chapter 11: Assessment 2]

Total: 60
Syllabus: General objective

This chapter deals with the application of ratio and enlargement/reduction on area and perimeter.

Specific objectives

• Solve problems involving practical applications of ratio on area and perimeter of two-dimensional shapes.
• Solve problems involving practical applications of scale factors of enlargement/reduction on area and perimeter of two-dimensional shapes.

In this chapter students will learn the skill to manipulate two-dimensional shapes with regard to their area and perimeter by changing their dimensions by a certain ratio or scale factor. It is important that students have a clear understanding of the effect that a change to dimensions of a shape has on the area and perimeter of the object.

Answers

Support Activity 12.1  
(Student’s Book page 209)

Area 1 (□) = l \times b = 14 \times 6 = 84 \text{ cm}^2
Area 2 (△) = \frac{1}{2}b \times h = \frac{1}{2} \times 4 \times 6 = 12 \text{ cm}^2
Area 3 (□) = \frac{1}{2}\pi r^2
= \frac{1}{2} \times \pi \times 16
= 25.13274122 \text{ cm}^2
Total Area = 84 + 12 + 25.13 = 121.13 \text{ cm}^2
Perimeter = 18 + 6 + 6 + \sqrt{4^2 + 6^2} + \frac{1}{2}(2\pi \times 4)
= 32 + \sqrt{52} + 4\pi
= 32 + 7.21 + 12.57
= 51.78 \text{ cm}
Exercise 12.1

1. a) Perimeter = \(2 \times 30 + 2 \times 60\)
   \[= 180 \text{ cm}\]
b) \(h_1 = \sqrt{\left(30^2 - 25^2\right)} = 16.58 \text{ cm}\)
   \(h_2 = \sqrt{\left(60^2 - 25^2\right)} = 54.54 \text{ cm}\)
   \[\therefore h_1 + h_2 = 71.12\]
Area = \(\frac{1}{2} (a \times b)\)
\[= \frac{1}{2} (50 \times 71.12)\]
\[= 1778 \text{ cm}^2\]
c) Length = 71.12 + 2(25) cm
\[= 121.12 \text{ cm}\]
d) and e) Cost = \(\frac{122}{100} \times 8.50 + \frac{1778}{10000} \times 12\)
\[= \text{P12.50}\]

Note: It is important for students to understand that a length of 121.12 cm will most likely be rounded off to 122 cm (in some cases hardware stores may cut only half-metre lengths and Thsepo might have to buy 1.5 metres, but for the purpose of this calculation we assume that he can buy 1.22 metres). Students must also realise the importance of converting the measurements to metres and m², and of course they must know how to do such conversions, i.e. why we divide by 100 in the one case but by 10 000 in the next.

2. a) Perimeter = \(2\pi r\)
   \[= 2\pi \times 10\]
   \[\approx 62.8 \text{ m}\]
b) Cost = 62.8 \times 55.50 = \text{P3 485.40}

Investigation

1. \(\frac{AC}{DF} = \frac{10}{5} = 2\)  \(\frac{AB}{DE} = \frac{6}{3} = 2\)  \(\frac{BC}{EF} = \frac{8}{4} = 2\)
   Therefore \(\frac{AC}{DF} = \frac{AB}{DE} = \frac{BC}{EF} = 2\)

2. Perimeter of triangle DEF = 12 cm and perimeter of ABC = 24 cm
   The ratio \(\frac{\text{perimeter } \triangle ABC}{\text{perimeter } \triangle DEF} = 2\). The ratios of sides and perimeters are the same.

3. Area of \(\triangle DEF = 6 \text{ cm}^2\) and area of \(\triangle ABC = 24 \text{ cm}^2\)
   The ratio \(\frac{\text{area } \triangle ABC}{\text{area } \triangle DEF} = \frac{24}{6} = 4 = 2^2\). The ratio of the area is the square of the ratio of the side lengths.
4. **Ratio of sides**

\[
\begin{align*}
\frac{RS}{DE} &= \frac{9}{3} = 3 \\
\frac{ST}{EF} &= \frac{12}{4} = 3 \\
\frac{RT}{DF} &= \frac{15}{5} = 3
\end{align*}
\]

**Ratio of perimeters**

\[
\begin{align*}
\frac{\text{perimeter } RST}{\text{perimeter of } DEF} &= \frac{36}{12} = 3
\end{align*}
\]

**Ratio of areas**

\[
\begin{align*}
\frac{\text{area of } RST}{\text{area of } DEF} &= \frac{54}{6} = 9
\end{align*}
\]

5. The ratios of the lengths of the sides are equal, because the triangles are similar. The ratios of the perimeters are the same, because the perimeters are the sum of the sides.

The ratio of the areas is the same, because the ratio of the perimeters are equal; and the ratio of the areas is a square, because length \times length is a square.

### Activity 12.1

(Student’s Book page 213)

1. **Perimeter of rectangle ABCD** = \(2(3 + 10) = 26\) cm

Area of rectangle ABCD is \(3 \times 10 = 30\) cm\(^2\)

Perimeter of MNOP = \(2(4.2 + 14) = 36.4\)

Area of MNOP = \(58.8\) cm\(^2\)

Ratio of length = \(\frac{14}{10} = \frac{7}{5}\)

Ratio of side = \(\frac{14}{10} = \frac{7}{5}\)  

Ratio of perimeters = \(\frac{36.4}{26} = \frac{364}{260} = \frac{7}{5}\)

Ratio of areas = \(\frac{58.8}{30} = \frac{588}{300} = \frac{49}{21} = \left(\frac{7}{5}\right)^2\)

2. a) **Triangles**  

Ratio of the sides is 1:3  

Ratio of perimeters is 1:3  

Ratio of areas is 1:9

b) **Rectangles**  

Ratio of sides is 2:3  

Ratio of perimeters is 2:3  

Ratio of areas is 4:9

c) **Arrows**  

Ratio of sides 7:4  

Ratio of perimeters is 7:4  

Ratio of areas is 49:16

### Extension Activity 12.1

(Student’s Book page 214)

Ratio of perimeters = 3:4, therefore ratio of areas is 9:16.

The sum of the areas is 75 cm\(^2\).

The area of the smaller triangle is \(75 \times \left(\frac{9}{25}\right) = 27\) cm\(^2\).

The area of the larger triangle is \(75 \times \left(\frac{16}{25}\right) = 48\) cm\(^2\).
Exercise 12.2

(Student's Book page 214)

1. a) 6:5 and 36:25
   b) 4:9 and 4:9

2. a) \( \frac{1}{2}bh \therefore h = \frac{\text{Area}}{\frac{1}{2}b} = \frac{60}{5} = 12 \text{ cm} \)
   b) \( h = \frac{2}{3} \times \frac{12}{1} = 8 \text{ cm}; b = \frac{2}{3} \times \frac{10}{1} = \frac{20}{3} = 6\frac{2}{3} \text{ cm} \)
       Area \( \triangle DEF = \frac{4}{9} \times \frac{60}{1} = \frac{240}{9} = 26\frac{2}{3} \text{ cm}^2 \)
   c) Area = \( \frac{1}{2}bh = \frac{1}{2} \times \frac{20}{3} \times \frac{8}{1} = \frac{160}{6} = 26\frac{4}{6} \)

3. a) \( \frac{9}{4} \)
   b) \( \frac{9}{4} \times \frac{100}{1} = 225\% \)
   c) Area 1: \( \frac{1}{2} \times 60 \times 40 = 1200 \text{ cm}^2 \)
       New dimensions: \( \frac{3}{2} \times 60 = 90 \text{ cm} \quad \frac{3}{2} \times 40 = 60 \text{ cm} \)
       Area 2: \( \frac{1}{2} \times 90 \times 60 = 2700 \text{ cm}^2 \)
       This is the same as \( 2.25 \times 1200 \).

4. a) \( \frac{BC}{FH} = \frac{DA}{GE} \)
   b) Ratio \( ABCD:EFHG = \frac{6}{4} = 3:2 \)
   c) \( x = \frac{2}{3} \therefore x = \frac{16}{3} \text{ cm} = 5.3 \text{ cm} \)
       \( y = \frac{3}{2} \therefore y = \frac{15}{2} = 7.5 \text{ cm} \)
   d) Perimeter \( ABCD:Perimeter \ EFHG = 3:2 \)
       Area \( ABCD:Area \ EFHG = 9:4 \)
   e) Area \( EFHG = \frac{4}{9} \times \frac{81}{1} = 36 \text{ cm}^2 \)

5. a) One window:
       Frame perimeter = \( 2(2 + 1.5) = 7 \text{ m} \)
       Length of slats = \( (4 \times 1.5) + (2 \times 2) = 10 \text{ m} \)

       **Cost per window:**
       Frame = \( 7 \times 20.50 = \text{P143.50} \)
       Slats = \( 10 \times 15.25 = \text{P152.50} \)
       Total = \( \text{P296/window} \)
       Cost of 9 windows: \( \text{P2 664} \)
   b) 4:25
   c) Cost = \( \frac{2}{5} \times 296 = \text{P118.40} \)
Extension Activity 12.2

(Student’s Book page 216)

Perimeter of ABCD = 50 cm  
Perimeter of A₁B₁C₁D₁ = 50 \times \left( \frac{5}{2} \right) = 125 \text{ cm}

Area of ABCD = 150 \text{ cm}^2 \text{ and Area of A₁B₁C₁D₁} = 150 \text{ cm}^2.

Ratio of perimeters is 5:2 and Ratio of area is 1:1.

The two triangles are not similar because the ratio of area is not a square of the ratio of perimeters.

Investigation

(Student’s Book page 216)

1. 

<table>
<thead>
<tr>
<th>Side</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>4 cm</td>
<td>1 cm²</td>
</tr>
<tr>
<td>2 cm</td>
<td>8 cm</td>
<td>4 cm²</td>
</tr>
<tr>
<td>3 cm</td>
<td>12 cm</td>
<td>9 cm²</td>
</tr>
<tr>
<td>4 cm</td>
<td>16 cm</td>
<td>16 cm²</td>
</tr>
<tr>
<td>5 cm</td>
<td>20 cm</td>
<td>25 cm²</td>
</tr>
<tr>
<td>6 cm</td>
<td>24 cm</td>
<td>36 cm²</td>
</tr>
</tbody>
</table>

a) Perimeter doubles in each case.
b) Area increases by a factor of 4.
c) \( \frac{\text{perimeter}}{\text{side}} = 4 \quad \frac{\text{area}}{\text{side}} = 1, \text{ then } 2, \text{ then } 3, \text{ then } 4, \text{ etc.} \) So the perimeter increases linearly and the area increases exponentially.
d) Tapiwa is right. If you increase the sides by a constant factor, the area increases by more than that constant factor.

2. 

<table>
<thead>
<tr>
<th>Side</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 cm by 4 cm</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>4 cm by 2 cm</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>2 cm by 1 cm</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

3. When you increase the dimensions by scale factor \( k \), the perimeter increases by scale factor \( k \) and area increases by scale factor \( k^2 \).

Activity 12.2

(Student’s Book page 218)

1. 2; 4
2. \( \frac{1}{5}, \frac{1}{25} \)
3. 4; 4
4. 2; 4
Extension Activity 12.3

For a square, if the side \( s = x \):
\[ P = 4x \quad A = x^2 \]
If the sides double, so \( s = 2x \):
\[ P = 4(2x) \quad A = (2x)^2 \]
For a rectangle with sides \( x \) and \( y \):
\[ P = 2(x + y) \quad A = xy \]
If the sides double, so \( 2x \) and \( 2y \):
\[ P = 4(x + y) \quad A = 4xy \]

Exercise 12.3

1. a) \( x = 4 \text{ cm}; \) perimeter = 16 cm
   b) Sides = 8 cm; scale factor 2
   c) 2 cm
2. a) Area = 144 mm\(^2\); perimeter = 48 mm
   b) i) Perimeter Y = \( 1.5 \times 48 = 72 \text{ mm} \)
      Area Y = \( 1.5^2 \times 144 = 324 \text{ mm}^2 \)
      ii) Perimeter Z = \( 1.5 \times 72 = 108 \text{ mm} \)
         Area Z = \( 1.5^2 \times 324 = 729 \text{ mm}^2 \)
   c) Side length of Y = \( 1.5 \times 12 = 18 \text{ mm} \)
      Side length of Z = \( 1.5^2 \times 12 \)
         = \( 1.5 \times 18 = 27 \text{ mm} \)
   d) \( 18^2 = 324 \text{ mm}^2; \) \( 27^2 = 729 \text{ mm}^2 \)
   e) \( 48:72:108 = 4:6:9 \)
   f) Area X:Area Y
      = 144:324
      = 1:2.25
      Area Y:Area Z
      = 324:729
      = 1:2.25
      X:Y:Z = 144:324:729 = 16:36:81
3. a) \( \frac{1}{25} \)
   b) \( 2\pi r; \frac{2}{5}\pi r; \frac{2}{25}\pi r; \frac{2}{125}\pi r \)
   Ratio = \( \frac{1}{5} \)

Exercise 12.4

1. a) Volume increases by a scale factor of 36. He will therefore use 360 ml of paint.
   b) Perimeter increases by a scale factor of 6. He will therefore use 6 times more braiding and it will cost him \( 6 \times 2.50 = P15.00 \).
2. a) \( \frac{1}{3} \times 60 = 15 \text{ m of fencing} \)
   b) \( \frac{1}{9} \times 450 = P50 \)
3. a) \(\frac{1}{1000}\)
   b) \(\frac{1}{1000}\)

4. a) The length and the width must each increase by a factor \(\sqrt{1.21} = 1.1\).
   So, width = 27.5 m; length = 35.2 m

**Project** *(Student’s Book page 221)*

1. 960 000 ha.
2. \(\frac{3}{178}\)
3. \(150 000 \times 1.045 = 156 750\)
4. 1 003 200
5. Percentage increase in land over five years is 1.045
6. 4.5%
7. It is not sustainable, as the elephants will be taking all the land in a few years to come. Elephants will have to be reduced, for example, relocated.

**Revision Exercise** *(Student’s Book page 222)*

1. a) \(\frac{5}{2}, \frac{25}{4}\)
   b) halve; be a quarter of the original.
   c) 4; 4
   d) 3:2; 9:4
2. a) 4 cm
   b) Area \(\triangle ABC = \frac{1}{2} \times 4 \times 3 = 6\) cm²
   c) \(\frac{1}{2.5^2} = \frac{1}{6.25}\)
   d) Area \(\triangle DEF = \text{Area } \triangle ABC \times 6.25\)
      \(= 6 \times 6.25\)
      \(= 37.5\) cm²
   e) \(DE = 2.5 \times 3 = 7.5\) cm
      \(EF = 2.5 \times 4 = 10\) cm
      \(DF = 2.5 \times 5 = 12.5\) cm
      Area \(\triangle DEF = \frac{1}{2} \times 10 \times 7.5 = 37.5\) cm²
3. a) Shaded area = \(18 \times 20 - \pi \times 9^2\)
      \(= 105.5\) m²
   b) Ratio = \(\frac{\pi \times 9^2}{105.5} = \frac{254.5}{105.5} = \frac{509}{211} = 2.4\)
   c) Shaded area = \(\frac{1}{4} \times 105.5 = 26.4\) m²
      Ratio = \(\frac{\pi \times 4.5^2}{26.4} = \frac{2.4}{1}\)
Note: It can be pointed out to students that the ratio stays the same since both areas changed by the same factor.

d) Shaded area = $18 \times 20 - 2 \times \pi \times 4.5^2$
   \[= 232.8 \text{ m}^2\]

Note: It is important for students to understand that two circles with a diameter of 9 metres each do not have the same area as one circle with diameter 18 metres, i.e. $2 \times 4.5^2 \neq 9^2$.

4. a) Shaded area = $\pi \times 40^2 - 2 \times \pi \times 20^2$
   \[= 2513.3 \text{ m}^2\]
   b) Ratio = $\frac{1}{1} = 1:1$ (i.e. the areas are equal)
   c) Perimeter small circles = $2(2 \times \pi \times 20)$
   \[= 251.33 \text{ m}\]
   Perimeter big circle = $2 \times \pi \times 40$
   \[= 251.33 \text{ m}\]
   Ratio = 1:1
   d) Diameters and perimeters double.

5. a) $\sqrt{(5^2 - 3^2)} = 4 \text{ cm}$
   b) Area = $\frac{1}{2} (\text{sum } \parallel \text{sides}) \times \perp \text{height}$
   \[= \frac{1}{2} (18 + 12) \times 4\]
   \[= 60 \text{ cm}^2\]
   c) Area = $3^2 \times 60 = 540 \text{ cm}^2$
   d) Ratio = 9:1

6. a) 0.75 m
   b) Area door = 3.9 m$^2$
      Perimeter door = 7.9 m
   c) $\frac{\text{Area window}}{\text{Area door}} = \frac{16}{25} = 0.64 : 1$
      $\frac{\text{Perimeter window}}{\text{Perimeter door}} = \frac{4}{5}$
   d) Area window = 2.5 m$^2$
      Perimeter window = 6.3 m
**Syllabus: General objective**

This chapter deals with applications of ratio, proportion, enlargement and reduction on surface area and volume of three-dimensional objects. It is important that students have a clear understanding of the effect that a change to dimensions of a 3-D object has on the surface area and volume of the object.

This chapter lends itself to teaching students the skill to manipulate three-dimensional objects with regard to their surface area and volume through changing their dimensions by a certain ratio or scale factor.

**Specific objectives**

- Solve problems that involve the application of ratio and proportion and scale factors of enlargement and reduction on surface area and volume of three-dimensional objects.

**Answers**

**Investigation** *(Student’s Book page 227)*

1. Surface area \( (SA_1) = 6 \times 1.2 \, \text{m} \times 1.2 \, \text{m} = 8.64 \, \text{m}^2 \)
2. Volume \( (V_1) = 1.2 \, \text{m} \times 1.2 \, \text{m} \times 1.2 \, \text{m} = 1.728 \, \text{m}^3 \)
3. Surface area \( (SA_2) = 6 \times 2.4 \, \text{m} \times 2.4 \, \text{m} \times 2.4 \, \text{m} = 34.56 \, \text{m}^2 \)
4. Volume \( (V_2) = 2.4 \, \text{m} \times 2.4 \, \text{m} \times 2.4 \, \text{m} = 13.824 \, \text{m}^3 \)
5. \( \frac{SA_2}{SA_1} = 4 \) and \( \frac{V_2}{V_1} = 8 \)
6. The surface area changed by a scale factor of 4 \( = 2^2 \) and the volume changed by a scale factor of 8 \( = 2^3 \).

**Exercise 13.1** *(Student’s Book page 229)*

1. a) \( V_a = 113.1 \, \text{cm}^3; \, SA_a = 113.1 \, \text{cm}^2 \)
   
   b) \( V_b = 3 \, 053.6 \, \text{cm}^3 \) (\( = 27 \times V_a \))
   \( SA_b = 1 \, 017.9 \, \text{cm}^2 \) (\( = 9 \times 5 \))
   
   c) \( \frac{V_b}{V_a} = 27 = 3^3 \)
   
   d) \( \frac{SA_b}{SA_a} = 9 = 3^2 \)
2. a) \( V_a = \pi \times 10^2 \times 30 = 9424.8 \text{ cm}^3 \)
\[ SA_a = 2\pi r^2 + 2\pi rh = 2513.3 \text{ cm}^2 \]
b) \( V_b = \frac{1}{8} \times V_a = 1178.1 \text{ cm}^3 \)
\[ SA_b = \frac{1}{4} \times SA_a = 628.3 \text{ cm}^2 \]
c) \( \frac{V_b}{V_a} = \frac{1}{8} \)
d) \( \frac{SA_b}{SA_a} = \frac{1}{4} \)

**Support Activity 13.1** *(Student’s Book page 229)*

1. 4:9; 8:27
2. 3:2; 27:8
3. 2:5; 4:25

**Activity 13.1** *(Student’s Book page 230)*

1. a) Diameters: \( \frac{8}{24} = \frac{1}{3} \)
   Surface areas: \( \frac{1}{9} \)
   Volumes: \( \frac{1}{27} \)

**Note:** In the formula \( V = \frac{1}{3}\pi r^2 h \), both \( r \) and \( h \) are multiplied by the scale factor \( \frac{1}{3} \), therefore the scale factor by which the volume decreases works out to be \( (\frac{1}{3})^2 \times \frac{1}{3} = \frac{1}{27} \).

b) and c) Truncated cone surface area
\[ = SA_{\text{full cone}} - SA_{\text{small cone}} + \pi \times 4^2 \]
\[ = 240\pi - \frac{1}{9} \times 240\pi + 16\pi \]
\[ = 229\frac{1}{3}\pi \text{ cm}^2 \left( \frac{8}{9} \times 240\pi + 16\pi \right) \]
\[ \therefore \text{Ratio} = \frac{229\frac{1}{3}\pi}{240\pi} = \frac{688}{720} = \frac{43}{45} \]

Truncated cone volume
\[ = V_{\text{full cone}} - V_{\text{small cone}} \]
\[ = 2304\pi - \frac{1}{27} \times 2304\pi \]
\[ = \frac{26}{27} \times 2304\pi \]
\[ = 2218\frac{2}{3}\pi \text{ cm}^3 \]
\[ \therefore \text{Ratio} = \frac{2218\frac{2}{3}\pi}{2304\pi} = \frac{6656}{6912} = \frac{26}{27} \]

OR:
\[ \text{Ratio} = 1 - \frac{1}{27} = \frac{26}{27} \]
Exercise 13.2

1. a) \( V = 324 \text{ cm}^3 \)
   \( SA = 342 \text{ cm}^2 \)
   b) \( \frac{SA_{\text{small}}}{SA_{\text{big}}} = \frac{4}{9}, \quad \frac{l_{\text{small}}}{l_{\text{big}}} = \frac{2}{3} \)
   c) \( V_{\text{small}} = \frac{8}{27} \times 324 = 96 \text{ cm}^3 \)
   \( SA_{\text{small}} = \frac{4}{9} \times 342 = 152 \text{ cm}^2 \)
   d) Dimensions
   \( = \left(\frac{2}{3} \times 3\right) \times \left(\frac{2}{3} \times 12\right) \times \left(\frac{2}{3} \times 9\right) \)
   \( = 2 \times 8 \times 6 \text{ cm} \)
   e) \( V = 96 \text{ cm}^3 \)
   \( SA = 2[(2 \times 8) + (2 \times 6) + (8 \times 6)] \)
   \( = 152 \text{ cm}^2 \)
2. a) \( V = 729 \text{ cm}^3; \quad SA = 6 \times 81 = 486 \text{ cm}^2 \)
   b) \( V = \left(\frac{4}{3}\right)^3 \times 729 = 1 728 \text{ cm}^3 \)
   \( SA = \left(\frac{4}{3}\right)^2 \times 486 = 864 \text{ cm}^2 \)
   c) Side \( = \frac{4}{3} \times 9 = 12 \text{ cm} \)
   \( \rightarrow 12 \text{ cm} \times 12 \text{ cm} \times 12 \text{ cm} \)
   d) \( V = 12^3 = 1 728 \text{ cm}^3 \)
   \( SA = 6 \times 12 \times 12 = 864 \text{ cm}^2 \)
3. a) \( \frac{H}{h} = \frac{3}{2} = \frac{R}{r} \)
   b) \( \frac{SA_{\text{big}}}{SA_{\text{small}}} = \frac{9}{4}, \quad \frac{V_{\text{big}}}{V_{\text{small}}} = \frac{27}{8} \)
   c) \( SA_{\text{big}} = \frac{9}{4} \times 400 = 900 \text{ cm}^2 \)
   d) \( V_{\text{small}} = \frac{8}{27} \times 108 = 32 \text{ cm}^3 \)

Investigation

1. If the sides increase by scale factor 3, the surface area will increase by scale factor 9. The area will increase by scale factor 27.
2. If you halve the sides, the surface area decreases by scale factor \( \frac{1}{4} \). The area decreases by scale factor \( \frac{1}{8} \).
3. Conclusion: The surface area increases or decreases by the square of the sides. The area decreases by the factor of the power of three of the side.
4. a) Side \( x \) Surface area is \( 6x^2 \) Volume is \( x^3 \)
   b) Side \( 3x \) Surface area \( 6(3x)^2 \) Volume is \( (3x)^3 \)
   c) Side \( \frac{1}{2}x \) Surface area \( 6\left(\frac{1}{2}x\right)^2 \) Volume is \( \left(\frac{1}{2}x\right)^3 \)
Activity 13.2 (Student’s Book page 233)

1. 4; 8
2. \( \frac{1}{2}; \frac{1}{8} \)
3. 3; 27

Exercise 13.3 (Student’s Book page 234)

1. a) \( SA = 2\pi r^2 + 2\pi rh \)
   \[ = 534.1 \text{ cm}^2 \]
   \[ V = 942.5 \text{ cm}^3 \]
   b) \( SA = 6 \times 8^2 = 384 \text{ cm}^2 \)
   \[ V = 83 = 512 \text{ cm}^3 \]
2. a) \( SA = 9 \times 534.1 = 4806.9 \text{ cm}^2 \)
   \[ V = 27 \times 942.5 = 25447.5 \text{ cm}^3 \]
   b) \( SA = 9 \times 384 = 3456 \text{ cm}^2 \)
   \[ V = 27 \times 512 = 13824 \text{ cm}^3 \]
3. a) \( SA = 4\pi r^2 = 314.2 \text{ cm}^2 = 0.0314 \text{ m}^2 \)
   b) Number of spheres = \( \frac{5}{0.0314} = 159 \)
   c) Scale factor = \( \left( \frac{1}{2} \right)^2 = \frac{1}{4} \)
   Number of spheres = \( 4 \times 159 = 636 \)
   
   Note: The actual answer is 636.9, but since 0.9 is not a full sphere, the number of spheres that can be fully covered by the available material is only 636.

4. a) \( V = 5301.4 \text{ m}^3 \)
   b) \( SA = \pi r^2 + 2\pi rh = 1590.4 \text{ m}^2 \)
   c) Cost = \( \frac{1590.4}{5} \times 10.75 = \text{P3 419.43} \)
   
   Note: Realistically, paint gets bought in 20-litre containers, especially for a job of this magnitude. It would therefore be more realistic to work out how many 20-litre containers are needed (\( \frac{1590.4}{5} \div 20 = 15.9 \approx 16 \) containers), and then calculate the cost of it, in this case \( 16 \times 20 \times 10.75 = \text{P3 440} \).
   d) i) \( V = \frac{1}{8} \times 5301.4 = 662.7 \text{ m}^3 \)
      ii) Cost = \( \frac{1}{4} \times 3 \times 419.43 = \text{P854.86 (P860)} \)

5. a) \( \frac{\text{volume } B}{\text{volume } A} = \frac{1024}{16} = 64 \)
   b) \( \frac{\text{length of base } B}{\text{length of base } A} = \text{cube root of 64} = 4 \)
   c) \( \frac{\text{surface area of } B}{\text{surface area of } A} = \text{square of length} = 16 \)
   d) Base area of pyramid B is \( 8 \times 16 = 128 \text{ cm}^2 \)
6. If the second block of wood is half the size, the cost of the second block will be \( P320 \times \left(\frac{1}{4}\right) = P80.00 \). He will need \( 1.2 \text{ litres} \times \left(\frac{1}{8}\right) = 0.15 \) litres of varnish.

### Extension Activity 13.1
(Student’s Book page 236)

1. a) \( s = \sqrt{12^2 + 6^2} = 13.4 \text{ cm} \)
   
   b) \( SA = 2\pi r^2 + \pi rs \)
   
   \[ = 2\pi \times 6^2 + \pi \times 6 \times 13.416407... \]
   
   \[ = 479.1 \text{ cm}^2 \]

2. \( SA = \frac{9}{4} \times 479.08800... = 1077.9 \text{ cm}^2 \)

### Project
(Student’s Book page 237)

<table>
<thead>
<tr>
<th>Diameter in metres</th>
<th>Height in metres</th>
<th>Volume in m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>1.96375</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2.3565</td>
</tr>
<tr>
<td>1.5</td>
<td>2.5</td>
<td>4.4184375</td>
</tr>
<tr>
<td>1.5</td>
<td>3</td>
<td>5.302125</td>
</tr>
<tr>
<td>2</td>
<td>1.75</td>
<td>5.4985</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6.284</td>
</tr>
</tbody>
</table>

1. 1.5 m in diameter and height 3 m is enough.
2. The tank with the smallest surface area is the one 1 metre in diameter and height 2.5 m.
3. \( SA = 3.142 \times 1 \times (0.5 + 2.5) = 9.426 \text{ m}^2 \).
   
   The amount of metal to the nearest metre will be 10 m².
   
   Cost: \( 10 \text{ m}^2 \times 180 + \frac{18}{100} \times 1800 = P2\ 124 \)

### Revision Exercise
(Student’s Book page 238)

1. a) 9:4; 27:8
   
   b) 2; 8

2. a) \( V = 162 \text{ cm}^3; SA = 198 \text{ cm}^2 \)
   
   b) 1:4; 1:2
   
   c) \( V = 20.25 \text{ cm}^3; SA = 49.5 \text{ cm}^2 \)

3. a) \( SA \text{ per ball} = 4\pi r^2 = 254.5 \text{ cm}^2 \)
   
   Leather required = 3 053.63 cm²

   b) Leather required = \( \left(\frac{2}{3}\right)^2 \times 3053.63 \)
   
   = 1 357.17 cm²

4. a) 1:25
   
   b) 1:5
   
   c) 9 cm
5. a) \(\frac{16}{49}\) and \(\frac{64}{343}\)
   b) i) 210 mm
       ii) \(\frac{4}{7} \times 210 = 120\) mm

6. Dimensions increase by a factor of \(\sqrt[3]{\frac{27}{8}} = \frac{3}{2}\)

7. a) \(V = (\pi R^2 - \pi r^2) \times h\)
    b) Cost = \(4.6 \times 150 = \text{P690}\)
    c) \(SA = 30.6\) m²
    d) Cost = \(\text{P78.95} \times 30.6 = \text{P2 415.87}\)
    e) He decreased the dimensions by factor \(\frac{2}{3}\).
    f) i) \(V = (\frac{2}{3})^3 \times 4.6 = 1.36\) m³ concrete
       ii) Savings = \(690 \times \frac{19}{27} = \text{P485.56}\)
           \((1 - (\frac{2}{3})^3 = \frac{19}{27})\)
           Note that cost = \((\frac{2}{3})^3 \times 690\)
           = \(\text{P204.44}\)
       iii) \(SA = (\frac{2}{3})^2 \times 30.6 = 13.6\) m²
       iv) Savings = \((1 - (\frac{2}{3})^2) \times 2415.87\)
           = \(\text{P1 342.15}\)
Syllabus: General objective

This chapter deals with applications of distance, time and speed to real-life situations.

Specific objectives

- Draw and interpret displacement-time graphs and solve problems involving these.
- Draw and interpret velocity-time graphs and solve problems involving these.

This chapter teaches students the skill to draw, interpret and use displacement-time and velocity-time graphs in real-life situations.

Answers

Support Activity 14.1 (Student’s Book page 245)

1. A. Gradient = 0; Velocity = 0 m/s
   Object stationary.

B. Gradient = constant and positive
   → Displacement uniform
   Velocity = positive, so the object moves at constant velocity away from the reference point, so in a straight line.

C. Gradient NOT constant → acceleration, so velocity increases with time and so displacement per second increases. Gradient gets steeper with time, BUT: it is still a CONSTANT acceleration, so there is even growth in steepness of graph.

D. Gradient NOT constant → acceleration (although negative), but the velocity decreases with time, so the displacement per second decreases. Gradient gets flatter with time, although it is still a CONSTANT deceleration.

Note: A negative acceleration is a deceleration.

2. a) A
   b) D
   c) B
   d) C
Activity 14.1  (Student’s Book page 248)

2. a) and b)

c) Phako has stopped to look at a flower bush.
d) 3 seconds.

Activity 14.2  (Student’s Book page 249)

1. a) \((0;0)\) is at P, the starting point. It then moves away from P, stops at B (turning point/maximum point on curve) comes back towards P (displacement decreases) until it goes past P (cuts time axis) in the opposite direction (below time axis) until it reaches D where it stops.
b) Displacement is positive above the time axis (when the ball moves between P and B) and negative below it (when the ball moves between P and D).
c) Velocity positive from P to B and negative from B back to P through to D.

Note: The velocity is zero at B since it is a turning point.

d) At B and D.
e) 3 m to the left of P.

2. It travels upwards while it slows down uniformly until it stops (at B); it then comes back at a uniform acceleration (under the influence of gravity) and moves past the point from where it was launched (at P) until it hits the floor (at D). No, the ball travels in a straight line that goes vertically upwards and then along the same line vertically downwards until it reaches the floor. The graph is a representation of the displacement of the ball relative to the reference point (P).

3. No. Displacement changes as time goes on and the horizontal axis is the time axis, so the graph is bound to 'follow the time' from left to right.
Activity 14.4

(Student’s Book page 252)

2. a) The ball goes up in the air to 3 metres above the point from where it was thrown (reference point) in 1 second, after which it stops and then falls back, past the reference point to a point 9 metres below the reference point in the next 2 seconds. The ball falls under a constant acceleration, which is why the curve has a parabolic shape (there is a quadratic relationship between the displacement and the time).

b) Displacement positive for $0 \leq t \leq 2$.

c) After 1 second (the velocity changes direction).

d) 3 m/s upwards.

Exercise 14.1

(Student’s Book page 253)

1. a) 1.5 metres

b) $\frac{1.5}{0.8} = 1.875$ s

b) $\frac{1.5}{0.8} = 1.875$ s

c) 3.375 s

d) 

e) Total displacement = 0 m

2. a) AB: Object stationary for 2 seconds at 1 metre from the reference point.

BC: Object accelerates at a constant rate (velocity increases) for 2 seconds and is displaced 3 metres from the starting point (4 metres from the reference point).

CD: Object moves at constant velocity (0.5 m/s) for 2 seconds and is displaced by another metre.

DE: Object moves back to the reference point at a constant velocity ($-5$ m/s) in 1 second.

b) $-1$ m

c) Distance $= 3 + 1 + 5 = 9$ m

d) Velocity $= \frac{\text{displacement}}{\text{time}} = \frac{-5}{1} = -5$ m/s

e) Average velocity $= -\frac{1}{7} - 0.14$ m/s
f) Object stopped and turned back on the same path.
g) Average velocity $= \frac{3}{2} = 1.5 \text{ m/s}$

3.

4. a) AB: Moves at constant velocity for 3 seconds, displaced 3 metres; stops at B.
BC: Turns back and moves at constant velocity for 2 seconds in opposite direction; displaced $-2$ metres.
CD: Moves at increased velocity until 1 metre past reference point, all in 1 second. Displaced $-2$ metres; stops at D.
DE: Turns back on same path and moves at constant velocity for 2 seconds, 1 metre past reference point. Displaced 2 metres.
b) Stopped and turned back at B and D. Accelerated at C.
c) $V_{AB} = 1 \text{ m/s in a certain direction}$
$V_{BC} = -1 \text{ m/s in the opposite direction}$
$V_{CD} = -2 \text{ m/s in the opposite direction}$
$V_{DE} = 1 \text{ m/s in the original direction}$
d) $V_{AB} = 3.6 \text{ km/h in a certain direction}$
$V_{BC} = -3.6 \text{ km/h; opposite direction}$
$V_{CD} = -7.2 \text{ km/h; opposite direction}$
$V_{DE} = 3.6 \text{ km/h in the original direction}$
e) i) 1 metre
  ii) 9 metres
  iii) Average speed $= \frac{\text{distance}}{\text{time}} = \frac{9}{8} = 1.125 \text{ m/s}$
f) At reference point A.
g) At 0 s and at 7 s.
Extension Activity 14.1

1.

2. a) Same for both, since they reach the end point at exactly the same time.
   b) Ra Sam: 14.3 m/s; 7 m/s; 4.7 m/s; 4 m/s; 3.3 m/s all in a northern direction.

   **Note:** Only the average velocity per 3-second interval can be calculated since Ra Sam’s velocity changes constantly (broken-line graph). The instantaneous velocity can be found at a given time by finding the gradient of the curve at that time. (This technique falls outside the syllabus for Form 3.)

   Ra Buthi: velocity \( \frac{100}{15} = 6.7 \text{ m/s North} \)
   He maintained a constant velocity (let’s assume it to be possible) from beginning to end.

   c) Ra Sam was at 43 m, 64 m, 78 m, 90 m and 100 m from the starting point. He covered 43 m in the first 3 seconds, then 21 m in the next 3 s, 14 m in the next, 12 m and then 10 m.
   Ra Buthi was at 20 m, 40 m, 60 m, 80 m and 100 m from the starting point, so he covered 20 metres every 3 seconds throughout the race.

d) No winner, they both finished at the same time.

3. a) 23 metres
   b) 25 metres
   c) 18 metres
   d) 10 metres
   e) 0 metres

4. a) Average velocity = \( \frac{100}{15} = 6.7 \text{ m/s North} \)
   b) Ra Sam: 14.3 m/s North;
      7 m/s North;
      4.7 m/s North; 4 m/s North;
      3.3 m/s North
   Ra Buthi: Velocity = \( \frac{100}{15} = 6.7 \text{ m/s North} \)

5. Graph (b)

6. No – they are too slow. Olympic athletes run 100 m in under 10 s.
Project

1. AB: Constant velocity for 2 minutes, away from A in a positive direction. Displacement is a straight line, which indicates a constant velocity. BC is where the object stands still, so the velocity is zero. CD is displacement in a direction opposite to the original direction, therefore it is a negative velocity (since it is in the opposite direction) but still at a constant velocity, hence the straight line for displacement. Note that the time on the velocity-time graph has been converted to hours in order to express the velocity in km/h.
2. The velocity stays constant, i.e. acceleration is zero.
3. Area A = 2 km
   Area B = −6 km
   The areas represent the displacement since km/h × h = km.
4. The displacement is zero in that time interval.
5. Yes, the object ends up 4 km from its starting point, but in a negative direction.

Activity 14.5

1. Velocity = 4 m/s East after 2 seconds
2. Velocity stays constant at 12 m/s East for 14 seconds.
3. The scooter stops.
4. 0 – 6 seconds
5. BC: 20 – 24 seconds
6. It accelerates constantly from a standstill for 6 seconds, then travels at a constant velocity (in a straight line) for 14 seconds, after which it slows down at a constant rate for 4 seconds until it stops.
7. 6 seconds
8. Displacement = area under velocity-time graph.
   ∴ Displacement = \(\frac{1}{2} \times 4 \times 8 = 16\) m
9. Total area = total displacement
   \(= \frac{1}{2} \times 6 \times 12 + 14 \times 12 + \frac{1}{2} \times 4 \times 12\)
   \(= 228\) metres
10. 222 metres
Exercise 14.2

(Student’s Book page 260)

1. a) An object moving at a constant (uniform) velocity, for example, a car on a straight road.
   b) An object slowing down at a uniform rate until it stops, for example, a scooter.
   c) An object slowing down at a constant rate until it comes to a standstill, and then it accelerates in the opposite direction at the same constant rate.
   d) An object slowing down at a non-uniform rate until it stops. A car, truck, train, etc.

2. a) 22.5 m; 67.5 m; 157.5 m
   b) The car slowed down uniformly.
   c) The car stopped.
   d) −13.5 m (13.5 m in direction opposite to the initial direction of movement)
   e) 157.5 m
   f) 0 m/s^2
   g) 

Extension Activity 14.2

(Student’s Book page 261)

1. 

Radius of orbit = \(1\,609 + 6\,400\)
   \(= 8\,009\) km

Distance = \(2\pi \times 8\,009 = 50\,322\) km

2. 0 km
3. Speed = $\frac{\text{distance}}{\text{time}}$
   
   $= \frac{50,322 \text{ km}}{118.41 \text{ hours}}$
   
   $= \frac{50,322 \text{ km}}{1.9735 \text{ h}}$
   
   $\approx 25,500 \text{ km/h}$

4. Velocity = 0 since displacement = 0

---

**Revision Exercise**

* (Student’s Book page 262)

1. a) 

   ![Graph Image]

   b) $\frac{3}{33} = \frac{1}{11} = 0.09 \text{ m/s away from } A.$

   c) $\frac{8}{80} = -0.1 \text{ m/s away from } A, \text{ i.e. } 0.1 \text{ m/s towards } A.$

   d) 

2. a) $\frac{3}{5} = 0.6 \text{ m/s}$

   b) 0 m/s
3. a) Velocity = $40 \times 3.6 = 144$ km/h
   No, it is not safe and it is over the speed limit anyway.

b) 

c) Distance = Area under velocity-time graph + 5 metres
   $\text{Distance} = 40 \times 15 + 5 \times 30 + \frac{1}{2}(5 \times 10) + 10 \times 30 + \frac{1}{2}(4 \times 30) + 5$
   $= 1140$ metres ($= 1.14$ km)

d) Distance = $40 \times 34 = 1360$ metres
   The car would have been 220 metres past the elephant.

4. a) A: The potato was launched.
   B: The potato reached its maximum height, after 2 seconds, where it stopped and turned back.
   C: The potato reached the point from where it was launched, after 4 seconds.

b) It starts at $v = 30$ m/s upwards, slows down to $v = 0$ m/s within 2 seconds and then accelerates uniformly in the opposite direction until it reaches $v = 30$ m/s downwards after 2 seconds (or $v = -30$ m/s upwards).
c)

d) 0 metres
e) 60 metres

f) Speed = \frac{\text{distance}}{\text{time}} = \frac{60}{4} = 15 \text{ m/s} = 54 \text{ km/h}
Syllabus: General objective

This chapter deals with knowledge on working with variables.

Specific objectives

• Expand and/or simplify algebraic expressions of up to 4 terms and of degree 3.
• Express as a single fraction, algebraic fractions with two terms and of degree 3, with both numerator and denominator having two terms.
• Factorise quadratic expressions and include the concept of difference of two squares.

The use of algebra and algebraic concepts play a huge role in problem solving in Mathematics, and the skills dealt with here form an integral part of the bigger picture in Mathematics. One often hears students ask: “... but where will I use it in life one day?”. This section of work lends itself to sketching the broader picture to students and to help them understand that Mathematics is like a tapestry from behind: it often seems like a lot of loose ends with no purpose, but when it eventually all comes together one day, it makes the most beautiful picture!

Answers

Activity 15.1 (Student’s Book page 266)

1. a) \(x^2; 6x; 24; 4x\)
   
   b) Total area = \(x^2 + 6x + 24 + 4x\)
   
   = \(x^2 + 10x + 24\) units\(^2\)

Exercise 15.1 (Student’s Book page 268)

1. a) \(-a + 2ab + 4b\)
   
   b) \(x^2 + 2x - 3\)
   
   c) \(x^2 - 5x - 14\)
   
   d) \(x^3 - 4x^2 + 3x\)
   
   e) \(x^3 + 2x^2 + 2x - 3\)
2. a) \( x^2 + 5x + 6 \)
   b) \( -3x^2 + 3 \)
   c) \( -2x^3 + x^2 + 6x - 3 \)
   d) \( -3a^3 - 3a^2 + 9a \)
   e) \( -3a^3 + 8a^2 - a - 6 \)
   f) \( 4pk - 8p - 12k + 24 \)
   g) \( 3x^3 + 15x^2 + 18x \)
   h) \( p^3 - 4p^2 + p + 6 \)

3. a) Perimeter = 10x + 4 cm
   b) Area = 6x^2 + 8x - 8 cm^2

---

**Extension Activity 15.1**  
(Student’s Book page 268)

1. \((a + b)^1 = a + b\)
   \((a + b)^2 = a^2 + 2ab + b^2\)
   \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\)
   \((a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\)

2.  

\[
\begin{array}{cccc}
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
\end{array}
\]

3. Add the two numbers above to get the next row:

\[
\begin{array}{ccccccc}
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 \\
1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 \\
1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \\
\end{array}
\]

---

**Support Activity 15.1**  
(Student’s Book page 271)

1. \(4(x + 2y) + a(x + 2y)\)
   \(= (x + 2y)(4 + a)\)

2. \(3a(1 - 2a) - 2b(1 - 2a)\)
   \(= (1 - 2a)(3a - 2b)\)

3. \(3xy(8x - 3y) - 2y(8x - 3y)\)
   \(= y(8x - 3y)(3x - 2)\)

4. \(6(2m + n) - 2n(2m + n)\)
   \(= 2(2m + n)(3 - n)\)
Exercise 15.2

(Student's Book page 271)

1. a) 2 
   b) 8x 
   c) 11x^2 
   d) 7mn 
   e) 8xy 
   f) 2x – m 
   g) 3c – 1 

2. a) 3(a + b) 
   b) 7mn(m^2n – 8) 
   c) (3c – 1)(2a – b) 
   d) (2r – 1)(3x + 2y) 
   e) (x + 5)(x + 3) 
   f) (x + 5)(2x + 3) 
   g) (x + 6)(x – 6) 
   h) x(x – 3) + 3(x – 3) 
   = (x + 3)(x – 3) 

3. a) 2(3 – y) – x(3 – y) 
   = (3 – y)(2 – x) 
   b) (x – 1)(x – 5) 
   c) x(x – 2) – 8(x – 2) 
   = (x – 2)(x – 8) 
   d) (x + 4)(x + 1) 
   e) x(x + 3) + 7(x + 3) 
   = (x + 3)(x + 7) 
   f) (2x – 3)(2x + 3) 
   g) a(x + 7)(x – 3) 
   h) 2x(2x + 3) – 7(2x + 3) 
   = (2x – 7)(2x + 3) 

Activity 15.2

(Student’s Book page 272)

1. 60 
2. 40 
3. 8a^4 
4. a^2(a + 1) 
5. a^2b^3 

Exercise 15.3

(Student’s Book page 273)

1. \( \frac{y}{5} \)
2. \[ \frac{3a + 4b}{ab} \]

3. \[ \frac{2xy - x}{6y^3} \]

4. \[ \frac{b + 3}{2b(1 - b)} \]

5. \[ \frac{-2b}{(b + 1)(b + 2)} \]

6. \[ \frac{-x + 8}{x(x - 2)} \]

**Exercise 15.4** *(Student’s Book page 274)*

1. a) \[ \frac{3x^3}{2z} \]
   b) \[ \frac{a^2}{2b} \]
   c) \[ \frac{1}{2a^2(3 - b)} \]
   d) \[ \frac{2}{3 - b} \]
   e) \[ \frac{1}{x} \]
   f) \[ \frac{a}{2} \]
   g) \[ \frac{-x(35x + 48)}{6x^2(x - 4)} \]

2. Perimeter = \[ 2(l + b) \]
   \[ = 2\left(\frac{12b}{a} + \frac{a^3}{8b}\right) \]
   \[ = 2\left(\frac{96b^2 + a^4}{8ab}\right) \]
   \[ = \frac{a^4 + 96b^2}{4ab} \]

Area = \[ l \times b \]
\[ = \frac{12b}{a} \times \frac{a^3}{8b} \]
\[ = \frac{3a^2}{2} \]

**Extension Activity 15.2** *(Student’s Book page 274)*

Smaller volumes: \[ x^2(x + 2) + x(x + 1)(x + 2) + x(x + 2)^2 + (x + 1)(x + 2)^2 \]
\[ = 4x^3 + 14x^2 + 14x + 4 \]

Big volume: \[ (2x + 1)(x + 2)(2x + 2) \]
\[ = 4x^3 + 14x^2 + 14x + 4 \]

This equals the sum of the smaller volumes.
Activity 15.3  
(Student’s Book page 275)

1. a) 1  
b) $x + 2$  
c) $-4x$  
d) $x - 1$  
e) $x - 5$  
f) $(x - 3)(x - 1)$  
g) $x^2 + 2x - 8$  
h) $(x - 4)(x + 1)$  
i) $(x + 4)(x - 1)$

Activity 15.4  
(Student’s Book page 277)

1. a) $x^2 + 3x + 3x + 9$  
b) $x^2 - 4x - 3x + 12$  
c) $x^2 - 5x + x - 5$  
d) $x^2 + 4x - 2x - 8$  
2. a) $(x + 3)(x + 3)$  
b) $(x - 4)(x - 3)$  
c) $(x - 5)(x + 1)$  
d) $(x + 4)(x - 2)$

Exercise 15.5  
(Student’s Book page 277)

1. $(x + 5)(x + 5)$  
2. $(x + 5)(x - 1)$  
3. $(y + 3)(y - 1)$  
4. $(x + 7)(x - 1)$  
5. $(x - 5)(x + 4)$  
6. $(x - 12)(x - 2)$  
7. $(k - 3)(k - 3)$  
8. $(x - 8)(x - 1)$  
9. $(x - 30)(x - 1)$

Exercise 15.6  
(Student’s Book page 278)

1. $(5x - 4)(x + 1)$  
2. $(3a + 1)(a + 8)$  
3. $(2d - 1)(d + 3)$  
4. $(7x - 3)(2x + 5)$  
5. $(4y - 5)(2y - 3)$
Activity 15.5  
(Student’s Book page 279)

6. a) \((x + 1)^2\)  
b) \((x + 2)^2\)  
c) \((x + 6)^2\)  
d) \((2x - 1)^2\)

Exercise 15.7  
(Student’s Book page 280)

1. a) \((a + 2)^2\)  
b) \((y + 7)^2\)  
c) \((x + y)^2\)  
d) \((b - 8)^2\)  
e) \((2x + y)^2\)  
2. a) \((k - 9)\)  
b) \((7a - 5b)\)  
c) \((3a + 2b)\)

Investigation  
(Student’s Book page 281)

1. a) \(AB = 3, BC = 4, AC = 5\)  
b) \(AB = 8, BC = 6, AC = 10\)  
c) \(AB = 5, BC = 12, AC = 13\)  
2. Yes. The sides satisfy the equation \(a^2 + b^2 = c^2\).  
3. For right-angled triangle ABC:  
\[AC^2 = AB^2 + BC^2\]  
\[= (m^2 - n^2)^2 + (2mn)^2\]  
\[= m^4 - 2m^2n^2 + n^4 + 4m^2n^2\]  
\[= m^4 - 2m^2n^2 + 4m^2n^2 + n^4\]  
\[= m^4 + 2m^2n^2 + n^4\]  
\[= (m^2 + n^2)^2\]  
\[= AC^2\]

Extension Activity 15.3  
(Student’s Book page 281)

Hypotenuse = \(\sqrt{(n^2 - 4)^2 + (4n)^2}\)  
\[= \sqrt{n^2 - 8n + 16 + 16n^2}\]  
\[= \sqrt{17n^2 + 8n + 16}\]
Exercise 15.8  
(Student’s Book page 283)

1. \((x - 9)(x + 9)\)
2. \((2x - 1)(2x + 1)\)
3. \((j - 13y)(j + 13y)\)
4. \((3m - 5)(3m + 5)\)
5. \(2(t - 2)(t + 2)\)
6. \(5m(m - 3n)(m + 3n)\)
7. \(2(6 - x)(6 + x)\)
8. \([5 - (b + 3)][5 + (b + 3)]\)
\[= (2 - b)(8 + b)\]
9. \((-3 - d)(9 - d)\)
10. \(3(5 - x)(5 + x)\)
11. \((4z - 3)(4z + 3)\)
12. \(3ab^2(9a^2 - 4)\)
\[= 3ab^2(3a - 2)(3a + 2)\]
13. \(3x^3y^5z(x - 4yz^2)(x + 4yz^2)\)

Exercise 15.9  
(Student’s Book page 284)

1. a) \(\frac{4b^2(1 + 2b)}{b + 1} \times \frac{(b - 1)(b + 1)}{b}\)
\[= 4b(1 + 2b)(b - 1)\]

b) \(4(b^2 - 3)\)

c) \((-2b + 3)(b - 1)
\[= \frac{3b}{3b - 3}\]
\[\text{or} \quad \frac{b^2 - 8b + 9}{-b(2b - 3)}\]

2. \(P = 2\left[\frac{(b + 1)(b + 3)}{3(b + 1)} + \frac{(b - 2)(b + 2)}{3(b + 2)}\right]\)
\[= 4b + 23\]

\(A = \frac{(b - 2)(b + 2)}{3(b + 3)} \times \frac{(b + 3)(b + 1)}{3(b + 1)}\)
\[= \frac{b^2 - 4}{9}\]

Revision Exercise  
(Student’s Book page 285)

1. a) \(b^2(b - 1) + 3b(b - 1)\)
\[= b(b - 1)(b + 3)\]

b) \(a^2 + a - 6\)

c) \(x^3 - 3x^2 - 10x\)

d) \(x^3 - 8x + 16\)

e) \(4a^2 + 12ab + 9b^2\)
Chapter 15: Working with variables

2. (a) \( \frac{2xy - x}{6y^3} \)
   
   (b) \( \frac{-2b}{(b + 1)(b + 2)} \)
   
   (c) \( \frac{x^2 + x - (x - 2)(x + 4)}{x(x - 2)} \)
   
   \( = \frac{-x + 8}{x(x - 2)} \)
   
   (d) \( \frac{3bc^2d}{2} \)
   
   (e) \( \frac{y}{2} \)
   
   (f) \( \frac{d - 3}{3} \)

3. (a) \( (p + q)^2 \)
   
   (b) \( (6x + 1)^2 \)
   
   (c) \( (t - 7)(t + 7) \)
   
   (d) \( [7 - (g + 6)][7 + (g + 6)] \)
   
   \( = (1 - g)(13 + g) \)
   
   (e) \( (x + 5)(x - 4) \)
   
   (f) \( 2(x - 2)(x - 1) \)
   
   (g) \( (3x + 1)(2x - 1) \)
   
   (h) \( (3x - 2)(2x + 3) \)
   
   (i) \( (x + 1)(2 + y) \)
   
   (j) \( (x - 1)(3x + 4y) \)
Syllabus: General objective
This chapter deals with knowledge of formulae.

Specific objectives
• Change the subject of a formula to a specified variable by involving the use of factorising, square roots and cube roots.
• Solve problems involving changing the subject of a formula as applied in practical situations.

Answers

Activity 16.1

1. a) Area of a circle
   b) Speed = \( \frac{\text{distance}}{\text{time}} \)
   c) Compound interest
   d) \( n^{\text{th}} \) term of arithmetic sequence
   e) Distance between points \((x_1;y_1)\) and \((x_2;y_2)\)

Extension Activity 16.1

1. \( V_{esc} = \left( \frac{2 \times 6.673 \times 10^{-20} \times 5.9742 \times 10^{24}}{6.371 \times 10^3} \right)^{\frac{1}{2}} \)
   = \((125.5739)^{\frac{1}{2}}\)
   = 11.19 km/s

3. \( V_{esc} = \left( \frac{2 \times 6.673 \times 10^{-20} \times 7.36 \times 10^{22}}{1.737 \times 10^9} \right)^{\frac{1}{2}} \)
   = 2.3780 km/s
   Factor = \( \frac{2.3780}{11.19} = 0.2 \)
Homework Activity  
(Student’s Book page 291)

Yes, –40 °C = –40 °F

Investigation  
(Student’s Book page 292)

1. 1, 3, 5, 7, 9, 11, 13, 15, 17, etc.
2. 4, 9, 16, 25, 36, etc.
3. The resulting sums are perfect squares.
4. \(n^2\), where \(n\) is the position of the number, e.g. the sum of the first three odd numbers is 9, which is \(3^2\).

Exercise 16.1  
(Student’s Book page 293)

1. \(h = \frac{V}{lb}\)
2. \(t = \frac{d}{S}\)
3. \(P = \frac{A}{(1 + t)^n}\)
4. \(n = \frac{T_n - b}{a}\)
5. \(x_2 = 2x - x_1\)

Exercise 16.2  
(Student’s Book page 295)

1. \(yz + 5z = 3k\)
   \[\therefore z(y + 5) = 3k\]
   \[\therefore z = \frac{3k}{y + 5}\]
2. \(x(5 + y + d - 2) = 1\)
   \[\therefore x = \frac{1}{5 + y + d - 2}\]
3. \(3d^2 = 3\nu\)
   \[\therefore d = \pm\sqrt{\nu}\]
4. \(t = \frac{E}{IR}\)
5. \(r = \sqrt{\frac{A}{\pi}} \quad (r \text{ always } > 0)\)
6. \(P = \frac{E}{Rt}\)
   \[I = \sqrt{\frac{E}{Rt}} \quad (\text{currently always } > 0)\]
7. \(\pi = \sqrt{\frac{a\cdot E^2}{4r}} \quad (\pi \text{ is a constant value, so it does not make sense in reality to make a constant the subject of a formula, but for the purpose of the exercise we can accept it.})\)
8. \( T^2 = A^2 \times 20d \)
   \[ \therefore d = \frac{T^2}{20A^2} \]
9. \( H^2 = 4\pi^2 \frac{L}{9.8} \)
   \[ \therefore L = \frac{9.8H^2}{4\pi^2} \]
10. \( r = \sqrt{\frac{P}{C}} - I \)
11. \( b = 3a(\sqrt{k} - x) \)

**Activity 16.3** *(Student’s Book page 296)*

1. a) \( 6 \text{ m}^3 \)
   
   b) \( V = lbh \)

   c) \( h = \frac{V}{lb} \)

   d) \( h = \frac{6}{1.2 \times 2} = 2.5 \text{ m} \)

2. \( h = 1500(26 - T) = 1500(26 + 52) = 117000 \text{ m} \)

**Extension Activity 16.2** *(Student’s Book page 297)*

1. Cost = \( 1.15 \times 10 + 3.52x \)

2. \( P = \frac{0.04 + 1.7y + 1.5hhd - 0.13rf - 2.1hr + 0.53gov - Q}{0.17} \)

**Exercise 16.3** *(Student’s Book page 297)*

1. a) \( p = \frac{2A}{q} \)

   b) \( p = 2 \times 100 = 8 \text{ cm} \)

2. \( R = \frac{E}{I} \)

   a) \( E = IR \)

   b) \( I = \frac{E}{R} \)

   c) \( E = 1.5 \times 25 = 37.5 \text{ volts} \)

   d) \( I = \frac{12}{15} = 0.8 \text{ amperes} \)

3. \( d = \sqrt{8h\left[R - \frac{h}{2}\right]} \)

4. \( v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{E}{600}} \)

5. a) Cost = \( 300 + 45p \)

   b) \( p = \frac{C - 300}{45} = \frac{560 - 300}{45} = 5.8 \)

   c) Kabelo can take 4 people plus himself (5 people in total).
6. a) \( A = 6x \, m^2 \)
   b) \( \frac{288}{12} = 24 \, m^2 \)
   c) \( x = 4 \, m \)
7. \( t = 4.1 \, s \)

**Revision Exercise**

(Students Book page 299)

1. a) \( v = \pm \sqrt{\frac{F_r}{m}} \)
   b) \( a = \sqrt{2(s-ut)} \)
   c) \( t = \sqrt{\frac{4\pi^2r}{a_c}} \)
   d) \( T^2 = \frac{4\pi^2L}{g} \)
      \[ \therefore g = \left( \frac{4\pi^2L}{T^2} \right) \]
   e) \( s = \sqrt{2gt} \)
   f) \( c^2 = \frac{E}{m} \)
      \[ \therefore m = \frac{E}{c^2} \]
   g) \( h = \frac{ab}{\sqrt{a^2 + b^2}} \)

2. a) \( n = \sqrt[4]{8(T_n - 3)} \)
   \[ = 2\sqrt[4]{T_n - 3} \]
   b) \( n = 2 \times \sqrt[3]{30 - 3} = 6 \), so it is the 6th term

3. \( b = \frac{v^2}{D + 3h} = \frac{(2.5)^2}{50 + 3 \times 1.5} = 0.115 \)

4. a) \( C = 50x + 25y \)
   b) \( 800 = 50 \times 8 + 25y \)
      \[ \therefore y = 16 \, \text{chickens} \]
**Assessment 3**  
*Chapters 12 – 16*

**Question 1**
1.1 Shaded area = 153.5 m² ✓
1.2 Unshaded area = 300 – 153.5 ✓
   = 146.5 m² ✓
1.3 Perimeter + circumference = 66.4 m ✓
   Cost = 66.4 × 25 ✓ = P1 660 ✓
1.4 a) Cost = \( \frac{1}{2} \times 66.4 \times 25 \) ✓ = P830 ✓
   b) Shaded area = \( \left( \frac{1}{2} \right)^2 \times 153.5 \) ✓
   = 38.4 m² ✓

**Question 2**
2.1 \( h = \sqrt{10^2 - 6^2} \) ✓ = 8 cm ✓
2.2 Area = 12 × 12 × 8 ✓ = 48 cm² ✓
2.3 Volume = area base × height
   = 48 × 20 ✓
   = 960 cm³ ✓
2.4 TSA = 220 × 10 + 12 × 20 + 2 × 48 ✓
   = 736 cm² ✓
2.5 \( V = 27 \times 960 \) ✓ = 25 920 cm³ ✓
   TSA = 9 × 736 ✓ = 6 624 cm² ✓

**Question 3**
3.1 The object moves 1 metre away from A to B (left) in 1 second ✓, stops for 1 second, moves 4 metres in one second to the right to D ✓ and then turns back and moves 2 metres left in 2 seconds to E. ✓
3.2 Displacement = 1 m to the right. ✓
3.3 \( v_{AB} = 1 \) m/s left; \( v_{BC} = 0 \) m/s ✓; \( v_{CD} = 4 \) m/s right ✓; \( v_{DE} = 1 \) m/s left ✓
3.4

```
  5  |  4  |  3  |  2  |  1  |
  ---------------
  2  |  3  |  4  |  5  |  6  |
  ---------------
  -1  |
  -2  |
  -3  |
```

**[14]**
<table>
<thead>
<tr>
<th>Question 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 $8x^3 + 64y^3$</td>
</tr>
<tr>
<td>4.2 $-18x^3 + 60x^2 - 50x$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question 5</th>
</tr>
</thead>
</table>
| 5.1 a) $\frac{3y^2}{2}$ ✓ ✓
| b) $\frac{3x + 5}{2}$ ✓ ✓ |
| 5.2 a) $(x + 2)(a - b)$ ✓ ✓ ✓
| b) $(x - 2y)(x + 2y)$ ✓ ✓
| c) $(2x + 3)(x - 4)$ ✓ ✓
| d) $(2x + y)(2x - y + 6)$ ✓ ✓ ✓ |
| 5.3 a) $x = 1$ or $x = -2$ ✓ ✓ ✓
| b) $x = 0$ or $x = 5$ ✓ ✓ ✓
| c) $2x^2 - 3x - 9 = 0$ ✓
| $\therefore (2x + 3)(x - 3) = 0$ ✓
| $x = -\frac{3}{2}$ or $x = 3$ ✓ ✓ |

<table>
<thead>
<tr>
<th>Question 6</th>
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</thead>
<tbody>
<tr>
<td>6.1 $g = \pm\sqrt{\frac{12t}{m}}$ ✓ ✓ ✓</td>
</tr>
</tbody>
</table>
| 6.2 $x = \frac{m}{3(a + b)}$ ✓ ✓

Total: 80
Syllabus: General objective
This chapter deals with the understanding and application of quadratic equations.

Specific objectives
• Solve quadratic equations by factorisation.
• Solve problems involving quadratic equations as applied in practical life situations.
• Use a spreadsheet to solve quadratic equations.

Answers

Support Activity 17.1 (Student’s Book page 305)
1. \( x = 15 \)
2. \( x = \frac{60}{y} \)
3. \( x = 0 \)
4. \( x \in \mathbb{R} \) if \( y = 0 \)
5. \( x \in \mathbb{R} \)
6. \( x = 0; x = 2 \)
7. \( x = 0; x = -1 \)
8. \( x^2 - 2x - 4 = 0 \)
   \[ \therefore x = \frac{2 \pm \sqrt{20}}{2} = 3.24 \text{ or } -1.24 \]
9. \( x(x - 1) = 0 \)
   \[ \therefore x = 0 \text{ or } x = 1 \]

Exercise 17.1 (Student’s Book page 306)
1. \( x = 1 \) or \( x = -2 \)
2. \( x = 1 \) or \( x = -7 \)
3. \( x = 5 \) or \( x = 2 \)
4. \( k = -7 \) or \( k = 5 \)
5. \( m = -\frac{11}{2} \) or \( m = 3 \)
6. \( x = -3 \) or \( x = \frac{3}{2} \)
7. \( j = -5 \)
8. \( w = \frac{7}{4} \)
9. \( n = \pm 8 \)
10. \( x = \frac{2}{5} \)

**Exercise 17.2**

(Student's Book page 307)

1. \((k - 4)(k - 2) = 0\)
   \[ \therefore k = 4 \text{ or } k = 2 \]
2. \((x + 1)(x + 4) = 0\)
   \[ \therefore x = -1 \text{ or } x = -4 \]
3. \((2m + 1)(m + 5) = 0\)
   \[ \therefore m = -\frac{1}{2} \text{ or } m = -5 \]
4. \((3a + 2)(a - 2) = 0\)
   \[ \therefore a = -\frac{2}{3} \text{ or } a = 2 \]
5. \((w - 11)(w + 11) = 0\)
   \[ \therefore w = \pm 11 \]

**Exercise 17.3**

(Student's Book page 308)

1. \(x^2 + 7x - 30 = 0\)
   \[ \therefore (x + 10)(x - 3) = 0 \]
   \[ \therefore x = -10 \text{ or } x = 3 \]
2. \(c^2 + 12c - 64 = 0\)
   \[ \therefore (c + 16)(c - 4) = 0 \]
   \[ \therefore c = -16 \text{ or } c = 4 \]
3. \(a^2 - 14a - 32 = 0\)
   \[ \therefore (a - 16)(a + 2) = 0 \]
   \[ \therefore a = 16 \text{ or } a = -2 \]
4. \(b^2 = 36\)
   \[ \therefore b = \pm \sqrt{36} = \pm 6 \]
5. \(x^2 - 49 = 0\)
   \[ \therefore (x - 7)(x + 7) = 0 \]
   \[ \therefore x = \pm 7 \]
6. \(y^2 - y - 12 = 0\)
   \[ \therefore (y - 4)(y + 3) = 0 \]
   \[ \therefore y = 4 \text{ or } y = -3 \]
7. \(g = 52\)
8. \(m(m - 5) = 0\)
   \[ \therefore m = 0; m = 5 \]
9. \( j^2 - 6j + 8 = 0 \)
\[ \therefore (j - 4)(j - 2) = 0 \]
\[ \therefore j = 4 \text{ or } j = 2 \]

**Extension Activity 17.1**  
(Student’s Book page 308)

1. Multiply each term by \( x \) first: \( 2x^2 = 2 - 3x \)
\[ \therefore 2x^2 + 3x - 2 = 0 \]
\[ \therefore (2x - 1)(x + 2) = 0 \]
\[ \therefore x = \frac{1}{2} \text{ or } x = -2 \]

2. Multiply each term by \( x \) first: \( 3x^2 - 3x = 2 + x^2 \)
\[ \therefore 3x^2 - 3x - 2 - x^2 = 0 \]
\[ \therefore 2x^2 - 3x - 2 = 0 \]
\[ \therefore (2x + 1)(x - 2) = 0 \]
\[ \therefore x = -\frac{1}{2} \text{ or } x = 2 \]

**Activity 17.2**  
(Student’s Book page 311)

1. \( l = x + 5 \)
2. \( x(x + 5) = 14 \)
3. \( x^2 + 5x - 14 = 0 \)
\[ \therefore (x + 7)(x - 2) = 0 \]
\[ \therefore x = -7 \text{ or } x = 2 \]
4. \( x = 2 \)
5. Substitute \( x = 2 \) into the first expression to get the length, so \( l = 2 + 5 = 7 \). Therefore width is 2 m and length is 7 m.

**Exercise 17.4**  
(Student’s Book page 311)

1. \( 7x + 2x^2 = 4 \)
\[ \therefore 2x^2 + 7x - 4 = 0 \]
\[ \therefore (2x - 1)(x + 4) = 0 \]
\[ \therefore x = \sqrt{2} \text{ or } x = -4 \]
The answer must be an integer, so the answer is \(-4\).

2. \( x(x - 5) = 24 \)
\[ \therefore x^2 - 5x - 24 = 0 \]
\[ \therefore (x - 8)(x + 3) = 0 \]
\[ \therefore x = 8 \text{ or } x = -3 \]
The length cannot be negative, so \( l = 8 \text{ cm and } w = 3 \text{ cm.} \)

3. \( x^2 - 23x = 210 \)
\[ \therefore x^2 - 23x - 210 = 0 \]
\[ (x - 30)(x + 7) = 0 \]
\[ x = 30 \text{ m (length)} \]
\[ \text{Width} = \frac{\text{area}}{\text{length}} = \frac{210}{30} = 7 \text{ m} \]

4. a)

\[ \begin{array}{c}
 \text{x} \\
 x + 1 \\
 x + 2 \\
 x \\
\end{array} \]

b) \( (x + 2)^2 = (x + 1)^2 + x^2 \)

c) \( x^2 + 4x + 4 = x^2 + 2x + 1 + x^2 \)
\[ \therefore 0 = x^2 - 2x - 3 \]
\[ \therefore x^2 - 2x - 3 = 0 \]
\[ \therefore (x - 3)(x + 1) = 0 \]
\[ \therefore x = 3 \text{ is the solution (measurements are positive)} \]
Measurements are: 3, 4 and 5 metres.

5. a) 48 cm²

c) \( (6 + x)(8 + x) = 96 \)
\[ \therefore x^2 + 8x + 6x + 48 - 96 = 0 \]
\[ \therefore x^2 + 14x - 48 = 0 \]
\[ \therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -14 \pm \sqrt{388}/2 \]
d) \( x \approx 2.85 \text{ cm to be added to each side.} \)

6. a) 65 m (when \( t = 0 \))

b) When \( h = 0 \) \( \therefore 5t^2 - 65 = 0 \)
\[ \therefore 5t^2 = 65 \]
\[ \therefore t^2 = 13 \]
\[ \therefore t = 3.6 \text{ s (only positive value is possible)} \]

7. a) 200

b) \( t^2 - 24t + 200 = 56 \)
\[ \therefore t^2 - 24t + 144 = 0 \]
\[ \therefore (t - 12)^2 = 0 \]
\[ \therefore t = 12 \text{ years} \]

---

**Exercise 17.5**

(Student’s Book page 314)

1. \( x = 0.73 \text{ or } x = -2.73 \)
2. \( x = -0.24 \text{ or } x = 4.24 \)
Project

(Student’s Book page 316)

1. \( d = 71.5 \text{ m} \)

2. \( 0.007v^2 + 0.015v - 20 = 0 \)
\[ \therefore v = 52.4 \text{ km/h} \] (for 20 metres, from formula)
\[ 0.007v^2 + 0.0015v - 10 = 0 \]
\[ \therefore v = 36.7 \text{ km/h} \] (for 10 metres)
\[ 0.007v^2 + 0.0015v - 5 = 0 \]
\[ \therefore v = 25.7 \text{ km/h} \] (for 5 metres)

Revision Exercise

(Student’s Book page 317)

1. a) \((x - 7)(x - 5) = 0\)
\[ \therefore x = 7 \text{ or } x = 5 \]
b) \((x + 10)(x - 3) = 0\)
\[ \therefore x = -10 \text{ or } x = 3 \]
c) \(y = 0 \text{ or } y = 6\)
d) \(b^2 - b - 20 = 0\)
\[ (b - 5)(b + 4) = 0 \]
\[ \therefore b = 5 \text{ or } b = -4 \]
e) \(h = \pm k\)
f) \(x^2 + 7x - 30 = 0\)
\[ \therefore (x + 10)(x - 3) = 0 \]
\[ \therefore x = -10 \text{ or } x = 3 \]
g) \(a = 16 \text{ or } a = -2\)
h) \(x = -5 \text{ or } x = 3\)

3. a)
\[
\begin{array}{c}
40 + 2x \\
\hline
20 + 2x \\
\hline
x
\end{array}
\]

b) \(l = 40 + 2x\)
\[ w = 20 + 2x \]
c) Area walkway
\[ = (40 + 2x)(20 + 2x) - 800 \]
\[ = 120x + 4x^2 \]
d) \(4x^2 + 120x = 544\)
\[ \therefore x^2 + 30x - 136 = 0 \]
\[ \therefore (x - 4)(x + 34) = 0 \]
\[ \therefore x = 4 \text{ m} \] (the answer can’t be negative)
4. \( \frac{1}{2}(2 + x)x = 24 \)
\[ \therefore x^2 + 2x - 48 = 0 \]
\[ \therefore (x + 8)(x - 6) = 0 \]
\[ \therefore x = 6 \text{ (only solution)} \]

Height = 6 m; Base = 8 m

5. a) \( x - 6 \)

b) 3 cm
c) \( V = (x - 6)^2 \times 3 \)
\[ = 3(x^2 - 12x + 36) \]
\[ \therefore 3x^2 - 36x + 108 = 48 \]
\[ \therefore x^2 - 12x + 36 = 16 \]
\[ \therefore x^2 - 12x + 20 = 0 \]
\[ \therefore (x - 10)(x - 2) = 0 \]
\[ \therefore x = 10 \]
d) Cardboard is 10 cm \( \times \) 10 cm
Syllabus: General objective
This chapter deals with the use of graphs in quadratic equations.

Specific objectives
• Use graphical methods to solve quadratic equations.
• Solve problems involving the use of graphical methods in quadratic equations as applied in practical situations.
• Use a spreadsheet to solve quadratic equations by a graphical method.

Answers

Support Activity 18.1  
(Student’s Book page 321)

<table>
<thead>
<tr>
<th>x</th>
<th>−4</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x²</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>5x</td>
<td>−20</td>
<td>−15</td>
<td>−10</td>
<td>−5</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>y = x² + 5x + 6</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

2. (−4;2)  (−3;0)  (−2;0)  (−1;2)  (0;6)  (1;12)  (2;20)

3.  

4. (−3;0) and (−2;0)
Chapter 18: Quadratic graphs

Activity 18.1  (Student’s Book page 324)

1–3. The graph cuts the x-axis at −2 and +2.

\[ y \]

\[ -5 \quad -2 \quad 0 \quad 2 \quad 5 \]

4. \[ 4 - x^2 = 0 \]
   \[ \therefore x^2 = 4 \]
   \[ \therefore x = \pm \sqrt{4} = \pm 2 \]

Activity 18.2  (Student’s Book page 325)

2. For \( a < 0 \): maximum function
   For \( a > 0 \): minimum function
3. Fold along the axis of symmetry.
4. The graph turns on the fold line (axis of symmetry).
5. The fold line is halfway between the x-intercepts.

Activity 18.3  (Student’s Book page 327)

1. It has a \( \cap \) shape, so a maximum function; AS: \( x = 1 \); turning point is at (1;4);
   x-intercepts: \( x = 3 \) and \( x = -1 \); y-intercept: (0;3).

2.
Extension Activity 18.1

1. (1; 4)
2. $x = 1$
3. Axis of symmetry
4. $(-1; 0); x = -1$

Exercise 18.1

1. a) Minimum function; $a = -2$
   
   $x$-intercepts: -3 and -1
   
   Turning point (-2; -1)
   
   $y$-intercept (0; 3)

   b) Minimum function; TP (-5; 0); AS: $x = -5$

   $y$-intercept (0; 25)

   c) Minimum function; TP (0; -9); AS: $x = 0$

   Roots: $x = \pm 3$; $y$-intercept (0; -9)

   d) Maximum function; TP (0; 1); AS: $x = 0$

   Roots: $x = \pm 1$; $y$-intercept (0; 1)

2. a) 

   ![Graph](image1)

   b) 

   ![Graph](image2)

   c) 

   ![Graph](image3)

   d) 

   ![Graph](image4)
Extension Activity 18.2

1. a) \( x = -2 \) or \(-1\)
   b) \( x = -3 \)
   c) No real roots
2. 2, 1 and 0
3. \( b^2 - 4ac = 0 \rightarrow \) one root 
   (turns on the x-axis)
   \( b^2 - 4ac < 0 \rightarrow \) no roots 
   (turns before x-axis)
   \( b^2 - 4ac > 0 \rightarrow \) two roots 
   (cuts x-axis at two points)
4. Can be 2 (maximum), 1 or zero (minimum).

Exercise 18.2

1. Solutions: \((-3; -6); (-2; -6)\)
2. Solutions: \((-2; 4); (1; 1)\)
3. Solutions: \((-0.83; 4); (4.83; 4)\) (the quadratic formula is needed for this)
4. Solutions: \((4; 16); (-4; 16)\)
5. Solutions: \((4; 12); (-3; 12)\)

Activity 18.4

Let the equation be \( y = k(x - 0)(x - 1) \).
The TP is at \((\frac{1}{2}; 2)\), so substitute this into the equation:
\[
\therefore 2 - k\left(\frac{1}{2} - 0\right)\left(\frac{1}{2} - 1\right)
= k\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)
= -\frac{1}{4}k
\]
\[\therefore k = -8 \]
The correct equation is \( y = -8x(x - 1) \)
**Investigation**

(Student’s Book page 331)

1. \(6 - x\)

3. \((1;5), (2;8), (3;9), (4;8), (5;5), (6;0)\)

4.–5.

6. \(x\)-intercepts: 0 and 6. It tells us that the area = 0 when \(x = 0\) or \(x = 6\).

7. AS: \(x = 3\)

8. \((3;9)\)

9. \(x = 3\)

10. 9 units²

**Activity 18.5**

(Student’s Book page 332)

1. There will be a minimum number of bacteria (it is a minimum function).

2.

4. Minimum at \(T = \frac{1}{2} \degree C\)

5. TP \((\frac{11}{2}; \frac{53}{4})\)
6. $5.75 \times 100 = 575$ bacteria

7. $T^2 - T + 6 > 8$
   \[ T^2 - T - 2 > 0 \]
   \[ (T - 2)(T + 1) > 0 \]
   \[ T < -1 \, ^\circ C \text{ or } T > 2 \, ^\circ C \]

**Exercise 18.3**  
*(Student’s Book page 332)*

1. a) $P = 100I - 5F$

   \[ y \]
   \[ 550 \]
   \[ 500 \]
   \[ 450 \]
   \[ 400 \]
   \[ 350 \]
   \[ 300 \]
   \[ 250 \]
   \[ 200 \]
   \[ 150 \]
   \[ 100 \]
   \[ 50 \]
   \[ 0 \]
   \[ 0 \]
   \[ 5 \]
   \[ 10 \]
   \[ 15 \]
   \[ 20 \]

   \[ x \]

   b) i) $I = \frac{-100}{-10} = 10$ amp
      ii) $P_{\text{max}} = 500$ W
   c) $P = 375$ W
   d) $100I - 5F = 20$
      \[ F - 20I + 4 = 0 \]
      \[ I = 19.8 \text{ amp or 0.2 amp (quadratic formula used)} \]

2. a) Length = $(x - 4)$ cm
   b) Cut-out lengths are 2 cm each.
   c) Volume = area base $\times$ height
   d) and e)

   \[ y \]
   \[ 55 \]
   \[ 50 \]
   \[ 45 \]
   \[ 40 \]
   \[ 35 \]
   \[ 30 \]
   \[ 25 \]
   \[ 20 \]
   \[ 15 \]
   \[ 10 \]
   \[ 5 \]
   \[ 0 \]
   \[ 0 \]
   \[ 1 \]
   \[ 2 \]
   \[ 3 \]
   \[ 4 \]
   \[ 5 \]
   \[ 6 \]
   \[ 7 \]
   \[ 8 \]
   \[ 9 \]
   \[ 10 \]

   f) $2(x - 4)^2 = 50$
      \[ \therefore x = 9 \text{ cm} \]

3. a) $(15;45)$
b) \( y = kx(x - 30) \)
   \[ \therefore 45 = k(15)(15 - 30) \]
   \[ \therefore k = -0.2 \]
   \[ \therefore y = -0.2x(x - 30) \text{ (third equation)} \]
   c) \( y = -0.2 \times 10(10 - 30) = 40 \text{ m} \)

**Exercise 18.4**  
(Student’s Book page 337)

1. a) \( x = -3 \) or \( x = -1 \)
   b) \( x = -5 \)
   c) \( x = \pm 3 \)
   d) \( x = \pm 1 \)

**Revision Exercise**  
(Student’s Book page 339)

1. a) \( x = 3 \) or \( x = -2 \)
   b) \( x = -6 \) or \( x = 2 \)
   c) No solution
   d) \( x = 5 \) or \( x = -3 \)
   e) No solution

3. a)

b) i) 12 m
   ii) After 12 seconds
   iii) \(-t^2 + 11t + 12 = 6 \)
   \[ \therefore t = \frac{11 \pm \sqrt{145}}{2} \text{ 11.5 seconds (t cannot be negative)} \]
   iv) Maximum height at \( t = 5.5 \text{ seconds} \text{ (axis of symmetry).} \)

   \[ \text{Max } h = -(5.5)^2 + 11(5.5) + 12 \]
   \[ = 42.25 \text{ m} \]
**Syllabus: General objective**
This chapter deals with understanding different ways of processing data and compiling presentations.

**Specific objectives**
- Present data in a cumulative frequency table.
- Draw a cumulative frequency curve.
- Estimate the median from a cumulative frequency curve.
- Interpret data from a cumulative frequency curve.

**Answers**

**Extension Activity 19.1** *(Student's Book page 345)*
6 children

**Support Activity 19.1** *(Student's Book page 345)*

2. a) 11 plants were of a height less or equal to 69 mm.
   b) 80–89 mm
   c) 60
   d) 42
   e)
<table>
<thead>
<tr>
<th>Height in mm</th>
<th>Frequency</th>
<th>Height in mm</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50–59</td>
<td>9</td>
<td>50–59</td>
<td>9</td>
</tr>
<tr>
<td>60–69</td>
<td>11</td>
<td>50–69</td>
<td>20</td>
</tr>
<tr>
<td>70–79</td>
<td>6</td>
<td>50–79</td>
<td>26</td>
</tr>
<tr>
<td>80–89</td>
<td>16</td>
<td>50–89</td>
<td>42</td>
</tr>
<tr>
<td>90–99</td>
<td>8</td>
<td>50–99</td>
<td>50</td>
</tr>
<tr>
<td>100 +</td>
<td>10</td>
<td>50–100 +</td>
<td>60</td>
</tr>
</tbody>
</table>
   
   f) 20 plants (of 60 plants) were of a height less than or equal to 69, which means they were all less than 70 mm.
Exercise 19.1

(Student’s Book page 346)

1. a) 3
   b) 23
   c) 100
   d) 

<table>
<thead>
<tr>
<th>Number of computers, ( n )</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( n \leq 1 )</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>( n \leq 2 )</td>
<td>23</td>
<td>63</td>
</tr>
<tr>
<td>( n \leq 3 )</td>
<td>14</td>
<td>77</td>
</tr>
<tr>
<td>( n \leq 4 )</td>
<td>18</td>
<td>95</td>
</tr>
<tr>
<td>( n \leq 5 )</td>
<td>5</td>
<td>100</td>
</tr>
</tbody>
</table>

e) 1

2. a) 12.9%
   b) 

<table>
<thead>
<tr>
<th>Number of days absent</th>
<th>Frequency</th>
<th>Number of days absent, ( n )</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
<td>0 ( n \leq 1 )</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>( n \leq 2 )</td>
<td>125</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
<td>( n \leq 3 )</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>68</td>
<td>( n \leq 4 )</td>
<td>318</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>( n \leq 5 )</td>
<td>458</td>
</tr>
<tr>
<td>5</td>
<td>86</td>
<td>( n \leq 6 )</td>
<td>544</td>
</tr>
<tr>
<td>6</td>
<td>34</td>
<td>( n \leq 7 )</td>
<td>578</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>( n \leq 8 )</td>
<td>618</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td></td>
<td>600</td>
</tr>
</tbody>
</table>

c) 458 students

3. a) 

<table>
<thead>
<tr>
<th>Weight (kg)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( w \leq 0.5 )</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0.5 &lt; ( w \leq 1.0 )</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>1.0 &lt; ( w \leq 1.5 )</td>
<td>23</td>
<td>35</td>
</tr>
<tr>
<td>1.5 &lt; ( w \leq 2.0 )</td>
<td>28</td>
<td>63</td>
</tr>
<tr>
<td>2.0 &lt; ( w \leq 2.5 )</td>
<td>25</td>
<td>88</td>
</tr>
<tr>
<td>2.5 &lt; ( w \leq 3.0 )</td>
<td>12</td>
<td>100</td>
</tr>
</tbody>
</table>

b) 63

c) 37%
Chapter 19: Processing data and presentations

4. a) 

<table>
<thead>
<tr>
<th>Age last birthday</th>
<th>Frequency</th>
<th>Age last birthday</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–19</td>
<td>28</td>
<td>0 &lt; a ≤ 19</td>
<td>28</td>
</tr>
<tr>
<td>20–39</td>
<td>45</td>
<td>20 &lt; a ≤ 39</td>
<td>73</td>
</tr>
<tr>
<td>40–59</td>
<td>30</td>
<td>40 &lt; a ≤ 59</td>
<td>103</td>
</tr>
<tr>
<td>60–79</td>
<td>7</td>
<td>60 &lt; a ≤ 79</td>
<td>110</td>
</tr>
</tbody>
</table>

b) 110 people
c) 20–39
d) 73 people

Homework Activity

(Activity’s Book page 348)

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6–10</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>11–15</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>16–20</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>21–25</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>26–30</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>31–35</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

\(x = 16\)

Activity 19.1

(Activity’s Book page 351)

1. 

<table>
<thead>
<tr>
<th>Distance</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–5</td>
<td>5</td>
</tr>
<tr>
<td>6–10</td>
<td>46</td>
</tr>
<tr>
<td>11–15</td>
<td>123</td>
</tr>
<tr>
<td>16–20</td>
<td>181</td>
</tr>
<tr>
<td>21–25</td>
<td>220</td>
</tr>
<tr>
<td>26–30</td>
<td>237</td>
</tr>
<tr>
<td>31–35</td>
<td>240</td>
</tr>
</tbody>
</table>

2. 181
3. 20
4. Table of ordered pairs:

<table>
<thead>
<tr>
<th>$x$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>5</td>
<td>46</td>
<td>123</td>
<td>181</td>
<td>220</td>
<td>237</td>
<td>240</td>
</tr>
</tbody>
</table>

5. **Distance travelled to cattleposts**

![Graph of distance travelled to cattleposts](image)

**Exercise 19.2** *(Student’s Book page 352)*

1. **Weight to the nearest kilogram**

<table>
<thead>
<tr>
<th>Weight (kg)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>8</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
</tr>
</tbody>
</table>

**Cumulative frequency curve for weight to the nearest kg**

![Graph of cumulative frequency curve](image)
2. a) 200
   b) 31–40 years was the most common age group, and 71–80 was the least common age group
   c) 129 people
   d) 

<table>
<thead>
<tr>
<th>Age upper boundary (years)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>30</td>
<td>71</td>
</tr>
<tr>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>182</td>
</tr>
<tr>
<td>60</td>
<td>197</td>
</tr>
<tr>
<td>70</td>
<td>199</td>
</tr>
<tr>
<td>80</td>
<td>200</td>
</tr>
</tbody>
</table>

Ages of people visiting a museum

3. a) i) 

<table>
<thead>
<tr>
<th>Number of drinks</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–4</td>
<td>3</td>
</tr>
<tr>
<td>5–9</td>
<td>4</td>
</tr>
<tr>
<td>10–14</td>
<td>8</td>
</tr>
<tr>
<td>15–19</td>
<td>12</td>
</tr>
<tr>
<td>20–24</td>
<td>15</td>
</tr>
<tr>
<td>25–29</td>
<td>12</td>
</tr>
<tr>
<td>30–34</td>
<td>4</td>
</tr>
<tr>
<td>35–39</td>
<td>1</td>
</tr>
</tbody>
</table>
ii) | Upper number of drinks | Cumulative frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>19</td>
<td>27</td>
</tr>
<tr>
<td>24</td>
<td>42</td>
</tr>
<tr>
<td>29</td>
<td>54</td>
</tr>
<tr>
<td>34</td>
<td>58</td>
</tr>
<tr>
<td>39</td>
<td>59</td>
</tr>
</tbody>
</table>

b) **Drinks sold at lunchtime in a school**

![Graph showing the cumulative frequency of drinks sold at lunchtime in a school]

4. | Marks   | Frequency | Mark upper limit | Cumulative frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6–10</td>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>11–15</td>
<td>1</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>16–20</td>
<td>3</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>21–25</td>
<td>4</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>26–30</td>
<td>4</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>31–35</td>
<td>6</td>
<td>35</td>
<td>18</td>
</tr>
<tr>
<td>36–40</td>
<td>3</td>
<td>40</td>
<td>21</td>
</tr>
<tr>
<td>41–45</td>
<td>5</td>
<td>45</td>
<td>26</td>
</tr>
<tr>
<td>46–50</td>
<td>4</td>
<td>50</td>
<td>30</td>
</tr>
</tbody>
</table>
Marks scored in a Mathematics test by Form 3 students

Activity 19.3  
(Students Book page 355)

1.

<table>
<thead>
<tr>
<th>Age last birthday</th>
<th>Frequency</th>
<th>Age last birthday</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–19</td>
<td>12</td>
<td>0–19</td>
<td>12</td>
</tr>
<tr>
<td>20–39</td>
<td>25</td>
<td>0–39</td>
<td>37</td>
</tr>
<tr>
<td>40–59</td>
<td>35</td>
<td>0–59</td>
<td>72</td>
</tr>
<tr>
<td>60–79</td>
<td>16</td>
<td>0–79</td>
<td>88</td>
</tr>
<tr>
<td>80–99</td>
<td>3</td>
<td>0–99</td>
<td>91</td>
</tr>
</tbody>
</table>
2. Median = middle value of number of people, so at 47:

\[
\text{Cumulative Frequency}
\]

The middle value (median age) is estimated at 44 years.

**Exercise 19.3**  
(Student’s Book page 356)

1. 6.1 kg, 34 years, 15–19 drinks; 30 marks

2. a) Spectators wanted to be on time for the start of the game so not many arrived too late or after the start.

<table>
<thead>
<tr>
<th>Minutes before the game started</th>
<th>Frequency</th>
<th>Lower boundary of class interval</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 ≥ T &gt; 50</td>
<td>35</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>50 ≥ T &gt; 40</td>
<td>62</td>
<td>40</td>
<td>97</td>
</tr>
<tr>
<td>40 ≥ T &gt; 30</td>
<td>95</td>
<td>30</td>
<td>192</td>
</tr>
<tr>
<td>30 ≥ T &gt; 20</td>
<td>120</td>
<td>20</td>
<td>312</td>
</tr>
<tr>
<td>20 ≥ T &gt; 10</td>
<td>54</td>
<td>10</td>
<td>366</td>
</tr>
<tr>
<td>10 ≥ T &gt; 0</td>
<td>34</td>
<td>0</td>
<td>400</td>
</tr>
</tbody>
</table>
c) Cumulative frequency of time of arrival by spectators before a game

![Cumulative frequency graph](image)

- Median: 30 minutes

**Exercise 19.4** *(Student’s Book page 358)*

1. a) 46 kg  
   b) 84 students  
   c) 44 kg
2. a) 55%  
   b) 65%  
   c) 30 students
Revision Exercise  
(Student’s Book page 360)

1. a)  
<table>
<thead>
<tr>
<th>Month</th>
<th>Money saved (Pula)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>February</td>
<td>400</td>
<td>700</td>
</tr>
<tr>
<td>March</td>
<td>600</td>
<td>1 300</td>
</tr>
<tr>
<td>April</td>
<td>900</td>
<td>2 200</td>
</tr>
<tr>
<td>May</td>
<td>450</td>
<td>2 650</td>
</tr>
<tr>
<td>June</td>
<td>750</td>
<td>3 400</td>
</tr>
</tbody>
</table>

b) P3 400  
c) 4 months  
d) He saved more money during the second half of the time, because P1 300 is less than half of P3 400 (after 3 months, end of March).

2. a)  
<table>
<thead>
<tr>
<th>Number of school days missed</th>
<th>Number of students</th>
<th>Upper boundary of number of school missed limits</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–5</td>
<td>460</td>
<td>5</td>
<td>460</td>
</tr>
<tr>
<td>6–10</td>
<td>740</td>
<td>10</td>
<td>1 200</td>
</tr>
<tr>
<td>11–15</td>
<td>392</td>
<td>15</td>
<td>1 592</td>
</tr>
<tr>
<td>16–20</td>
<td>205</td>
<td>20</td>
<td>1 797</td>
</tr>
<tr>
<td>21–25</td>
<td>95</td>
<td>25</td>
<td>1 892</td>
</tr>
<tr>
<td>26–30</td>
<td>66</td>
<td>30</td>
<td>1 958</td>
</tr>
<tr>
<td>31–35</td>
<td>64</td>
<td>35</td>
<td>2 022</td>
</tr>
<tr>
<td>36–40</td>
<td>8</td>
<td>40</td>
<td>2 030</td>
</tr>
</tbody>
</table>

b)  
Number of days missed by students due to HIV/AIDS
c) About 1 800 students

d) Out of 2 030 students who missed full days of school, most missed less than the median days, so a small percentage of students missed more than the median days due to HIV/AIDS-related illness. We also see that 230 students missed a lot of school due to HIV/AIDS-related illness.

3.

![Marks of 210 students in a test](image)

a) 210 candidates
b) Estimated median mark is 53%.
c) 70 students
d) 45% is the pass mark (since 150 students scored above 45%)
**Syllabus: General objective**

This chapter deals with knowledge on measures of central tendency and measures of dispersion.

**Specific objectives**

- Find the modal class, median class, and mean class from a frequency table comprising of grouped data.
- Find modal class and mean class from a histogram comprising of grouped data.
- Calculate range and inter-quartile range from a distribution and cumulative frequency curve.
- Estimate quartiles and percentiles from a cumulative frequency curve.

**Answers**

### Exercise 20.1

1. a) i) Modal class is 61–70.
   
   ii) Median class is 61–70.

   iii) | Percentage mark | Frequency | Middle value | Middle value × frequency |
        |-------------|-----------|--------------|--------------------|
        | 31–40       | 3         | 35.5         | 106.5              |
        | 41–50       | 4         | 45.5         | 182                |
        | 51–60       | 16        | 55.5         | 888                |
        | 61–70       | 26        | 65.5         | 1 703              |
        | 71–80       | 14        | 75.5         | 1 057              |
        | 81–90       | 8         | 85.5         | 684                |
        | 91–100      | 9         | 95.5         | 859.5              |
        | Total       | 80        |              | 5 480              |

   Mean = \( \frac{5 480}{80} = 68.5 \)

   So the mean class is 61–70.

   b) 16 + 26 + 14 + 8 + 9 = 73 students

2. a) i) 138–142
   
   ii) 138–142
### Activity 20.2

1. The value in the middle position is 16.
2. It is the median of the values to the left of 16.
3. It is the median of the values to the right of 16.
4. The range $= 20 - 12$
   $$= 8$$
5. The inter-quartile range $= 18.5 - 13.5$
   $$= 5$$

### Activity 20.3

1. Inter-quartile values are like medians in that they occupy the middle position when arranged in order in the lower and in the upper half of the distribution.
   Lower quartile is the median of the lower half of the distribution and upper quartile is the median of the upper half of the distribution.
2. a) $\frac{n + 1}{2} = \frac{101}{2} = 50.5$. Estimated median weight for this value is 46 kg.
   b) The lower quartile value is 41. The upper quartile value is 54.
   c) Inter-quartile range is $54 - 41 = 13$. (Note: mark according to the students’ answers in (c)).
1. **Marks of 210 students in a test**

<table>
<thead>
<tr>
<th>Marks %</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>60</td>
<td>250</td>
</tr>
<tr>
<td>70</td>
<td>300</td>
</tr>
<tr>
<td>80</td>
<td>350</td>
</tr>
<tr>
<td>90</td>
<td>400</td>
</tr>
<tr>
<td>100</td>
<td>450</td>
</tr>
</tbody>
</table>

   a) 43%
   b) 65%
   c) 65 - 43 = 22

2. **Distance travelled by students to school**

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td>25</td>
<td>200</td>
</tr>
<tr>
<td>30</td>
<td>250</td>
</tr>
<tr>
<td>35</td>
<td>300</td>
</tr>
</tbody>
</table>

   a) Median = 15 km
   b) Lower quartile = 11.5 km
   c) Upper quartile = 20 km
   d) Inter-quartile range = 20 - 11.5 = 8.5 km
1. **Weights of 100 students**

   a) i) 46 kg    
       ii) 40 kg   
       iii) 54 kg 
       iv) $54 - 40 = 14$ 
   b) i) 23rd percentile = 39 kg 
        ii) 60th percentile = 48 kg 
        iii) 89th percentile = 64 kg 
   c) About 87%

2. **Distance travelled by students to school**

   a) i) 15 km 
       ii) 11.5 km 
       iii) 20 km 
       iv) $20 - 11.5 = 8.5$ km 
   b) i) 10 km 
        ii) 22 km
Revision Exercise

(Student’s Book page 377)

1. Mean is 6.933  Mode is 10  Median is 8

2. a)  

<table>
<thead>
<tr>
<th>Amount in Pula</th>
<th>Middle value</th>
<th>Frequency</th>
<th>Middle value × Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0 – P1 000</td>
<td>P500</td>
<td>3</td>
<td>P500 × 3 = P1 500</td>
</tr>
<tr>
<td>P1 001 – P2 000</td>
<td>P1 500.50</td>
<td>3</td>
<td>P1 500.50 × 3 = P4 501.50</td>
</tr>
<tr>
<td>P2 001 – P3 000</td>
<td>P2 500.50</td>
<td>9</td>
<td>P2 500.5 × 9 = P22 504.50</td>
</tr>
<tr>
<td>P3 001 – P4 000</td>
<td>P3 500.50</td>
<td>18</td>
<td>P3 500.50 × 18 = P63 009</td>
</tr>
<tr>
<td>P4 001 – P5 000</td>
<td>P4 500.50</td>
<td>13</td>
<td>P4 500.5 × 13 = P58 506.50</td>
</tr>
<tr>
<td>P5 001 – P6 000</td>
<td>P5 500.50</td>
<td>10</td>
<td>P5 500.50 × 10 = P55 005</td>
</tr>
<tr>
<td>P6 001 – P7 000</td>
<td>P6 500.50</td>
<td>2</td>
<td>P6 500.50 × 2 = P13 001</td>
</tr>
<tr>
<td>P7 001 – P8 000</td>
<td>P7 500.50</td>
<td>2</td>
<td>P7 500.5 × 2 = P15 001</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>60</strong></td>
<td></td>
<td><strong>P233 028.50</strong></td>
</tr>
</tbody>
</table>

b)  
i) Modal class is P3 001 – P4 000
ii) Median class is P3 001 – P4 000
iii) Mean \( \frac{P233\,028.50}{60} = P3\,883.80 \). Mean class is P3 001 – P4 000.

3. a) Median value is P19 600
b) Range of the distribution is P36 000 – P11 500 = P24 500
c) Lower quartile is P14 600.
d) Upper quartile is the middle number in the upper half of this distribution.
e) Inter-quartile range is P27 000 – P14 600 – P12 400.
Syllabus: General objective

This chapter deals with the understanding and application of scatter graphs in practical situations.

Specific objectives

- Use a spreadsheet to draw scatter graph.
- Use a spreadsheet to draw a line of best fit on a scatter graph.

Answers

Exercise 21.1

(Student’s Book page 382)

1. **Length against mass of a certain plant’s leaves**

![Graph showing length against mass of a certain plant's leaves.]
2. Height against weight for 12 patients in a clinic

<table>
<thead>
<tr>
<th>Weight (kg)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>154</td>
</tr>
<tr>
<td>58</td>
<td>156</td>
</tr>
<tr>
<td>59</td>
<td>158</td>
</tr>
<tr>
<td>60</td>
<td>160</td>
</tr>
<tr>
<td>61</td>
<td>162</td>
</tr>
<tr>
<td>62</td>
<td>164</td>
</tr>
<tr>
<td>63</td>
<td>166</td>
</tr>
</tbody>
</table>

3. Mother’s height against son’s height

<table>
<thead>
<tr>
<th>Son’s height (cm)</th>
<th>Mother’s height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>200</td>
<td>150</td>
</tr>
</tbody>
</table>
Exercise 21.2

1. **Length against mass of a certain plant’s leaves**

   - Mass (g)
   - Length (mm)
   - Data points plotted on a scatter graph showing a positive correlation.

2. **Height against weight for 12 patients in a clinic**

   - Weight (kg)
   - Height (cm)
   - Data points plotted on a scatter graph showing a weak correlation.

(Student’s Book page 385)
3. **Mother’s height against son’s height**

<table>
<thead>
<tr>
<th>Son’s height (cm)</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mother’s height (cm)</td>
<td>0</td>
<td>20</td>
<td>60</td>
<td>120</td>
<td>160</td>
</tr>
</tbody>
</table>

### Revision Exercise
*(Student’s Book page 387)*

1. **Hours worked against hours watching sports**

<table>
<thead>
<tr>
<th>Hours watching sports</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. a) Water used against average growth

Water used against average growth

b) There is a strong, positive correlation, i.e. more water used resulted in more growth.

3. a) Goals scored against goals conceded by 12 teams

Goals scored against goals conceded by 12 teams

b) There is a weak, negative correlation.
Syllabus: General objective
This chapter deals with the understanding and application of probability tables on real-life situations.

Specific objectives
- Represent possible outcomes of two events involving real-life situations using probability tables/sample space diagrams.
- Use probability tables/sample space diagrams to calculate the probabilities of two events involving real-life situations.

Answers

Activity 22.1  
(Student’s Book page 392)

1. Green spinner  
   3 | (1,3)  (2,3)  (3,3)  
   2 | (1,2)  (2,2)  (3,2)  
   1 | (1,1)  (2,1)  (3,1)  

   Red spinner  
   1  2  3

   There are 9 possible outcomes.

2. Coin 1  
   H  (H, H)  (H, T)  
   T  (T, H)  (T, T)  
   H  T

   Coin 2  
   There are 4 possible outcomes.
Exercise 22.1
(\textit{Student's Book page 392})

1. Using SU for Scripture Union, TD for Traditional dance, MC for Mathematics Club, CH for Chess, GC for Guidance and Counselling, MU for Music, DM for Drama, AT for Art, and DB for Debating:

\begin{tabular}{|c|c|c|c|c|c|}
\hline
SU & (SU, GC) & (SU, MU) & (SU, DM) & (SU, AT) & (SU, DB) \\
\hline
TD & (TD, GC) & (TD, MU) & (TD, DM) & (TD, AT) & (TD, DB) \\
\hline
MC & (MC, GC) & (MC, MU) & (MC, DM) & (MC, AT) & (MC, DB) \\
\hline
CH & (CH, GC) & (CH, MU) & (CH, DM) & (CH, AT) & (CH, DB) \\
\hline
\end{tabular}

\textbf{Monday}

\begin{tabular}{|c|c|c|c|c|}
\hline
GC & MU & DM & AT & DB \\
\hline
\end{tabular}

\textbf{Wednesday}

2. a) Taking TE for Tea, JU for Juice, WA for Water, ML for Milk, FD for Fizzy drink, BS for Biscuits, CK for Cake, and MP for meat pie:

\begin{tabular}{|c|c|c|c|}
\hline
Drink & TE & (TE, BS) & (TE, CK) & (TE, MP) \\
\hline
JU & (JU, BS) & (JU, CK) & (JU, MP) \\
\hline
WA & (WA, BS) & (WA, CK) & (WA, MP) \\
\hline
ML & (ML, BS) & (ML, CK) & (ML, MP) \\
\hline
FD & (FD, BS) & (FD, CK) & (FD, MP) \\
\hline
\end{tabular}

\textbf{b)} There are 15 possible outcomes.

3. a) A
\begin{tabular}{|c|c|c|}
\hline
Consonants & (A, R) & (A, L) & (A, D) \\
\hline
E & (E, R) & (E, L) & (E, D) \\
\hline
I & (I, R) & (I, L) & (I, D) \\
\hline
\end{tabular}

\textbf{b)} 9 choices

Activity 22.2
(\textit{Student's Book page 394})

\begin{tabular}{|c|c|c|c|}
\hline
Chicken & chips & salad \\
\hline
Beef & chips & salad \\
\hline
Vegetarian & chips & salad \\
\hline
\end{tabular}
The possible combinations are:
Chicken and chips
Chicken and salad
Beef and chips
Beef and salad
Vegetarian and chips
Vegetarian and salad

**Activity 22.3** *(Student’s Book page 395)*

1. The first slice and the second slice eaten by the child.
2. Because the same slice cannot be taken twice (once eaten it cannot be put back on the plate).
3. There are 20 possible choices.
4. 6 choices
5. a) $\frac{3}{10}$  
   b) $\frac{1}{10}$  
   c) $\frac{3}{5}$

**Activity 22.4** *(Student’s Book page 397)*

1. [Diagram]

2. a) 0  
   b) $\frac{1}{3}$  
   c) $\frac{2}{3}$

**Exercise 22.2** *(Student’s Book page 397)*

1. a) $\frac{1}{20}$  
   b) $\frac{2}{5}$  
   c) $\frac{1}{5}$
2. a) $\frac{2}{5}$  
   b) $\frac{2}{12}$
3. a) 

\[
\begin{array}{cccccccc}
& \text{H} & \text{T} & \text{H} & \text{H} & \text{H} & \text{T} & \text{T} & \text{T} & \text{T} & \text{H} & \text{T} & \text{T} & \text{H} & \text{T} \\
0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{array}
\]

b) i) 0.125  ii) 0.125  iii) 0.875

**Revision Exercise**

(Student’s Book page 399)

1. a) 

\[
\begin{array}{cccccccccccc}
G_5 & R_3 G_5 & R_2 G_5 & R_1 G_5 & G_1 G_5 & G_2 G_5 & G_3 G_5 & G_4 G_5 & G_5 G_5 \\
G_4 & R_3 G_4 & R_2 G_4 & R_1 G_4 & G_1 G_4 & G_2 G_4 & G_3 G_4 & G_4 G_4 & G_5 G_4 \\
G_3 & R_3 G_3 & R_2 G_3 & R_1 G_3 & G_1 G_3 & G_2 G_3 & G_3 G_3 & G_4 G_3 & G_5 G_3 \\
G_2 & R_3 G_2 & R_2 G_2 & R_1 G_2 & G_1 G_2 & G_2 G_2 & G_3 G_2 & G_4 G_2 & G_5 G_2 \\
G_1 & R_3 G_1 & R_2 G_1 & R_1 G_1 & G_1 G_1 & G_2 G_1 & G_3 G_1 & G_4 G_1 & G_5 G_1 \\
R_1 & R_3 R_1 & R_2 R_1 & G_1 R_1 & G_2 R_1 & G_3 R_1 & G_4 R_1 & G_5 R_1 & G_5 R_1 \\
R_2 & R_3 R_2 & R_2 R_2 & G_1 R_2 & G_2 R_2 & G_3 R_2 & G_4 R_2 & G_5 R_2 & G_5 R_2 \\
R_3 & R_3 R_3 & R_2 R_3 & R_1 R_3 & G_1 R_3 & G_2 R_3 & G_3 R_3 & G_4 R_3 & G_5 R_3 \\
R_3 & R_2 R_3 & R_1 R_3 & G_1 R_3 & G_2 R_3 & G_3 R_3 & G_4 R_3 & G_5 R_3 & G_5 R_3 \\
\end{array}
\]

b) 

\[
\begin{array}{cccc}
G & \frac{4}{7} & R & \frac{3}{7} \\
\frac{5}{8} & G & \frac{3}{7} & R \\
\frac{3}{8} & R & \frac{5}{7} & R \\
\frac{2}{7} & & & \end{array}
\]

c) 56

d) i) \frac{3}{21}  ii) \frac{2}{7}  iii) \frac{15}{28}
2. a) 

<table>
<thead>
<tr>
<th>dice</th>
<th>disc 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
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<tr>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

b) i) $\frac{7}{12}$    ii) $\frac{7}{12}$    iii) $\frac{3}{4}$

3. a) 

[Diagram of probability tree with branch fractions]
Syllabus: General objective
This chapter deals with the application of knowledge, skills and processes learnt to identify and solve real-life problems.

Specific objectives
- Gather data to support or refute the conjectures/hypothesis.
- Analyse and interpret data.
- Generalise results.
- Evaluate the product.

Answers

Activity 23.3
(Student’s Book page 406)
1. Pie chart or bar chart
2. Pie chart
3. Bar chart

Activity 23.4
(Student’s Book page 407)
1. a) South Africa and Mauritius followed the world trend: slight increase from 1995–2000, then remained constant.
   Tanzania and Botswana: sharp increase from 1995–2000, then remained constant. Outperformed the world trend.
   Uganda had a continuous increase from 1995 until 2002. Outperformed the world trend.
   Zimbabwe and Kenya: industry deteriorated, below world trend.

b) South Africa and Mauritius both have well-developed tourism industries that are running almost at capacity, i.e. there is not much room for further growth.
   Botswana and Tanzania both have fast-growing tourism industries that are not at full capacity yet, so there’s still a lot of growth possible, and they utilise it.
   Uganda’s tourism industry is growing rapidly and they use all the means that are available to develop it further.
Zimbabwe and Kenya are losing tourists for various reasons, and students can discuss the possible reasons why these two African countries are going backwards in their tourism industries while the rest of the African countries studied here are growing.

2. a) Just below 8%; 8%.
   b) July & October highest; May lowest.
   c) South Africa has a higher visitor rate in the summer months, probably due to its long coastal line. Many tourists visit the South African beaches. Botswana has a higher visitor rate in June-July, the winter months, when their climate is far more moderate than South Africa’s.

Exercise 23.1  
(Student’s Book page 409)

2. The infant mortality rate was still on the increase from 2000 to 2004 (62–70/1 000), but since then there was a sharp decrease from about 70 per 1000 to only 15 per thousand. It might be due to improved health care in the rural areas; better after-birth care; young mothers being educated on caring for their infants; better community involvement in looking after the babies; decline in teenagers falling pregnant.

3. More teenage mothers in rural areas than urban areas; an increase in teenage motherhood from 1971 to 1988, and there after a significant decrease to 2006. There are half as many urban mothers as there are rural mothers. Reason: might be that the urban teenagers are more street-wise or that they are more careful by not having unprotected sex, better protected by their social groups that they move in, whereas the rural teenagers might be more exposed to the risks of unhealthy relationships which might contribute to early pregnancies.
Assessment 4

Question 1
1.1 a) Two widths = 2a and length = b; fourth side of pen is formed by the river. ✓ (1)
   b) \( b = 100 - 2a \) ✓ (2)
   c) \( a(100 - 2a) = 800 \) ✓ (2)
   :. \( 2a^2 - 100a + 800 = 0 \) ✓ (2)
   :. \( a = 40 \) (not applicable) ✓ or \( a = 10 \) ✓ (2)
   :. So \( a = 10 \) and \( b = 80 \) ✓ (2)
   d) Dimensions: 80 metres long, 10 metres wide. ✓ ✓ (2)

1.2 a) \( (x - 5)(x + 1) = 0 \) ✓ (3)
   :. \( x = 5 \) ✓ or \( x = -1 \) ✓ (2)
   b) \( x = 5 \) metres is where the water jet hits the ground. ✓ ✓ (2)
   c) \( 5 \) m ✓ (2)
   d) \( 2 \) m ✓ (2)
   e) \( 9 \) m (substitute \( x = 2 \) into \( y = -x^2 + 4x + y \)) ✓ (1)

Question 2
2.1 a) 26.7 kg ✓ (2)
   b) \( Q_1 = 30 \) kg ✓ ✓; \( Q_3 = 22 \) kg ✓ ✓ (4)
   c) 3rd: 14.5 kg ✓ ✓; 10th: 18 kg ✓ ✓; 80th: 31.5 kg ✓ ✓ (6)
   d) 75 lambs are below 37 kg. ✓ ✓ (2)
   e) 15 heaviest lambs: 32 – 40 kg ✓ ✓ (2)

2.2 a) Classes Frequency (\( f \)) Class midpoints (\( x \)) \( f \times x \)

<table>
<thead>
<tr>
<th>Classes</th>
<th>Frequency (( f ))</th>
<th>Class midpoints (( x ))</th>
<th>( f \times x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10 &lt; x \leq 15 )</td>
<td>3</td>
<td>12.5</td>
<td>37.5</td>
</tr>
<tr>
<td>( 15 &lt; x \leq 20 )</td>
<td>11</td>
<td>17.5</td>
<td>192.5</td>
</tr>
<tr>
<td>( 20 &lt; x \leq 25 )</td>
<td>17</td>
<td>22.5</td>
<td>382.5</td>
</tr>
<tr>
<td>( 25 &lt; x \leq 30 )</td>
<td>28</td>
<td>27.5</td>
<td>770.0</td>
</tr>
<tr>
<td>( 30 &lt; x \leq 35 )</td>
<td>12</td>
<td>32.5</td>
<td>390.0</td>
</tr>
<tr>
<td>( 35 &lt; x \leq 40 )</td>
<td>9</td>
<td>37.5</td>
<td>337.5</td>
</tr>
</tbody>
</table>

\[ \Sigma f = n = 80 \] ✓ ✓ (4)
\[ \Sigma fx = 2110 \] ✓ ✓ (4)

b) Model class: \( 25 < x < 30 \) ✓ ✓ (2)

c) Median class: \( 25 < x < 30 \) ✓ ✓ (2)

d) Estimated mean = \( \frac{\Sigma fx}{n} \)
\[ \begin{align*}
&= \frac{2110}{80} \\
&= 26.375
\end{align*} \] ✓ ✓ (2)

Chapter 23: Research skills: Data analysis and interpretation
e) It is based on class mid-values, which is a rough approximation of the average for every class interval. It is the mean obtained for grouped data and is not based on the raw data.

**Question 3**

3.1

3.2 \( y = 26 + 0.62x \) ✓✓

3.3 Strong ✓ \((r = 0.81)\) ✓

3.4 Language proficiency enhances Mathematical proficiency for obvious reasons: if students understand what they read, and they can interpret questions well, then they can also find a way to solve Mathematical problems. ✓✓

**Question 4**

5.1 a) Student could have drawn EITHER a probability table OR a tree diagram

<table>
<thead>
<tr>
<th>Coin</th>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((H, 1))</td>
<td>((T, 1))</td>
</tr>
<tr>
<td></td>
<td>((H, 2))</td>
<td>((T, 2))</td>
</tr>
<tr>
<td></td>
<td>((H, 3))</td>
<td>((T, 3))</td>
</tr>
<tr>
<td></td>
<td>((H, 4))</td>
<td>((T, 4))</td>
</tr>
<tr>
<td></td>
<td>((H, 5))</td>
<td>((T, 5))</td>
</tr>
<tr>
<td></td>
<td>((H, 6))</td>
<td>((T, 6))</td>
</tr>
</tbody>
</table>

1 2 3 4 5 6

Die ✓✓✓
5.2 
a) Student could have drawn EITHER a probability table OR a tree diagram. 
The outcomes are:
R1; R2; R3; R4; R5; R6
B1; B2; B3; B4; B5; B6
Y1; Y2; Y3; Y4; Y5; Y6 \rightarrow 18 outcomes

b) \( \frac{2}{12} = \frac{1}{6} \) \( \checkmark \) \( \checkmark \) (3)

\[ \frac{Y1 + Y3 + Y5}{18} = \frac{3}{18} = \frac{1}{6} \] \( \checkmark \) \( \checkmark \) (2)

Total: 80
# Sample Examination Paper 1 Answers: Mathematics

## Multiple-choice questions

**Answers**

<p>| | | | | |</p>
<table>
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<th></th>
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<tbody>
<tr>
<td>B</td>
<td>B</td>
<td>C</td>
<td>B</td>
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<tr>
<td>B</td>
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<tr>
<td>B</td>
<td>B</td>
<td>B</td>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

*Marks: 40*
Sample Examination Paper 2 Answers: Mathematics

SECTION A

Question 1
1.1 4.8 ✓ ✓ ✓
1.2 ✓ ✓ (Segments correct)
   ✓ ✓ (Labels correct)

Station E
120°
Station A
45°
Station C
90°
Station D
30°
Station B
75°

Question 2
2.1 21 ✓
2.2 Triangle ✓

Question 3
3.1 90 ✓
3.2 (Line of best fit) ✓

Number of accidents

Years

[2]
4.1 10.5 cm ✓✓
4.2 DE ✓
4.3 44° ✓

Question 5
5.1 \( c = \frac{n(n+1)}{2n} \) ✓
5.2 P32.50 ✓✓

Question 6
6.1 \( 6d^2 + 22d + 12 \) ✓
6.2 2 ✓

Question 7
7.1 South Africa ✓
7.2 P817.22 (to two d.p.) ✓
7.3 P837.65 ✓

Question 8
\( y = x + 1 \) ✓

Question 9
9.1 P22 000 ✓
9.2 P7 000 ✓

Question 10
20.8 km/h ✓✓
**Question 11**

11.1 Figure L is the reflection of K under the line $m$. ✓

![Diagram of reflection](image1)

11.2 ✓

**Question 12**

12.1 (*60° angle correct*) ✓

12.2 (*Bisection correct*) ✓
SECTION B

Question 13
13.1 ✓✓✓ (3)
13.2 384 750 m² ✓✓ (4)
13.3 3 415.9 m ✓✓ (3)
13.4 86 rolls ✓✓ (2)

[12]

Question 14
14.1 110 ✓ (1)
14.2 1976, 1977 ✓ (1)
14.3 45 cases in 1977 ✓✓ (2)
14.4 42.6 ✓✓✓ (3)

[7]

Question 15
15.1 10 m/s ✓✓ (1)
15.2 2 seconds ✓ (1)
15.3 36 km/h ✓✓ (1)

[3]

Question 16
16.1 448 cm² ✓✓ (2)
16.2 480 cm³ ✓✓ (2)
16.3 Surface area is halved, so 224 cm² ✓; volume divided by 4 = 120 cm³ ✓ (2)

[6]

Question 17
17.1 ✓✓ (2)
17.2 4.33 m ✓✓✓ (3)

[5]
18.1

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>4</td>
<td>1 $\checkmark$</td>
<td>0 $\checkmark$</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$x^2 - 2$</td>
<td>2</td>
<td>-1 $\checkmark$</td>
<td>-2 $\checkmark$</td>
<td>-1 $\checkmark$</td>
<td>2</td>
</tr>
</tbody>
</table>

(5)

18.2 (Coordinates and parabola correct) $\checkmark$ $\checkmark$ (2)

18.3 (0; -2) $\checkmark$ $\checkmark$ (2)

18.4 (2;2) $\checkmark$ (-2;2) $\checkmark$ (2)

**Question 19**
The church looks like this in 3-D.
Plan of the church:

SECTION C

**Question 19**
19.1 \( T = 100 \) ✓ (1)
\[ P = 30 \text{ units} \] ✓ (1)
19.2 12 units ✓ (1)
19.3 \( T = n^2 \) ✓ (1)
\[ P = 3n \] ✓ (1)
19.4 \( n = \frac{P}{3} \) when \( T = 9 \) ✓
\[ \therefore n = 3 \] ✓ (2)

**Question 20**
20.1 \( y = \frac{x}{5} \) ✓ (1)

20.2
Question 21
21.1 ✓ (1)
21.2 132 rpm ✓✓ (2)

Question 22
22.1

22.2

[3]

Question 23

[4]